COMPITO B

ES. 1

$P(T)=0,3 \qquad P(A)=0,7 \qquad P(S|T)=0,75 \qquad P(S|A)=0,6$

i) $S=$ "arrivo entro 2 ore"

$$\boxed{P(S)} = P(S|T)\,P(T) + P(S|A)\,P(A) = 0,75 \cdot 0,3 + 0,6 \cdot 0,7 = \boxed{0,645}$$

ii) $$\boxed{P(T|S)} = \frac{P(S|T)\,P(T)}{P(S)}$$

per la formula di Bayes

$$= \frac{0,75 \cdot 0,3}{0,645} = \boxed{0,35}$$

ES. 2

$X_i =$ "n° pallina estratta all'i-esima estraz."

i) $\boxed{P(X_2 > X_1)} = P(X_1 > X_2) = \boxed{\frac{1}{2}}$ per simmetria

ii) $$\boxed{P(X_2 > X_1 \mid \text{I è V})} = \frac{P(X_2 > X_1 \cap \text{I° pall. è V})}{P(\text{I pall. V})}$$

$$= \frac{\binom{30}{2} - \binom{20}{2} \Big/ \cancel{\binom{40}{2}}}{39 \cdot 10 \Big/ \cancel{\binom{40}{2}}} = \frac{\frac{30 \cdot 29}{2} - \frac{20 \cdot 19}{2}}{39 \cdot 10} = \boxed{0,63}$$

iii) $\boxed{F \sim Geom(p)}$ con $p = P(R) = \frac{1}{4}$

IN ALTERNATIVA SI VEDA ULTIMA PAGINA

$$\Rightarrow P(T=j) = \frac{1}{4}\left(\frac{3}{4}\right)^{j-1} \qquad j=1,2,\dots$$

2

$$\boxed{P(T\le 3)} = \sum_{j=1}^{3} \frac{1}{4}\left(\frac{3}{4}\right)^{j-1}$$

$$= \frac{1}{4}\sum_{\ell=0}^{2}\left(\frac{3}{4}\right)^{\ell} = \cancel{\frac{1}{4}}\,\frac{1-\left(\frac{3}{4}\right)^{3}}{\cancel{1-\frac{3}{4}}} = 1-\frac{27}{64}$$

$$= \boxed{\frac{37}{64}}$$

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B) EJ. 3

3

i) $P(X>Y) = \int_0^{+\infty} P(X>y)\, f_Y(y)\, dy$

$= \mu \int_0^{+\infty} e^{-y} e^{-\mu y}\, dy = \frac{\mu}{1+\mu}$

ii) $X^{1/n} \in (0,+\infty)$ p.c.

$F_{X^{1/n}}(z) = \begin{cases} 0 & z \leq 0 \\ ? & z > 0 \end{cases}$

$F_{X^{1/n}}(z) = P(X^{1/n} < z) = P(X < z^n)$

$= 1 - e^{-z^n} \qquad z \geq 0$

$\Rightarrow F_{X^{1/n}}(z) = \begin{cases} 0 & z \leq 0 \\ 1 - e^{-z^n} & z > 0 \end{cases} \xrightarrow{n \to +\infty} \begin{cases} 0 & z \leq 0 \\ 0 & 0 < z \leq 1 \\ 1 & z > 1 \end{cases}$

$\Rightarrow \boxed{X^{1/n} \xrightarrow[P]{d} 1}$

iii) per $n = 2$

calcolo $f_{X^{1/2}}(z) = \frac{d}{dz} F_{X^{1/2}}(z) = 2z\, e^{-z^2} \mathbb{1}_{z \geq 0}$

$\mathbb{E}(X^{1/2}) = 2 \int_0^{+\infty} z^2 e^{-z^2}\, dz$

$= \frac{\cancel{2}}{\cancel{2}} \int_0^{+\infty} \frac{w}{\sqrt{w}} e^{-w}\, dw$

$= \Gamma\left(\frac{3}{2}\right) = \frac{\sqrt{\pi}}{2}$

$w = z^2$

~~$dw = 2z\,dz$~~

$z = \sqrt{w}$

$dz = \frac{1}{2\sqrt{w}}$

$$\mathbb{E}((X^{1/2})^2) = \mathbb{E}X = \frac{1}{\lambda} = 1 \quad \text{poiché } \lambda = 1$$ (4)

$$\Rightarrow \frac{\sum_{i=1}^{n} X_i^{1/2} - \frac{n\sqrt{\pi}}{2}}{\sqrt{n\left(1-\frac{\pi}{4}\right)}} \xrightarrow{d} Z \sim N(0,1)$$ per il TLC

poiché $V(X) = 1 - \frac{\pi}{4}$

$$\Rightarrow \boxed{\frac{\sum_{i=1}^{n} X_i^{1/2} - \frac{n\sqrt{\pi}}{2}}{\sqrt{n}} \xrightarrow{d} W \sim N\left(0, 1-\frac{\pi}{4}\right)}$$

poiché se $Z \sim N(0,1)$ e $W = aZ$
$\Rightarrow W \sim N(0, a^2)$

Es. 1

ii) in ALTERNATIVA

$\boxed{P(X_2 > X_1 \mid I^a\ V)} = P(A)$ esito II^a estr.

$= P(A|R)P(R) + P(A|V)P(V) + P(A|G)P(G) +$
$+ P(A|B)P(B)$

$= 0 \cdot \frac{10}{38} + \frac{1}{2} \cdot \frac{8}{38} + 1 \cdot \frac{10}{38} + 1 \cdot \frac{10}{38} = \frac{20}{38} + \frac{8}{76} = \frac{48}{78}$

$= \boxed{0{,}63}$ come calcolato a p. 1