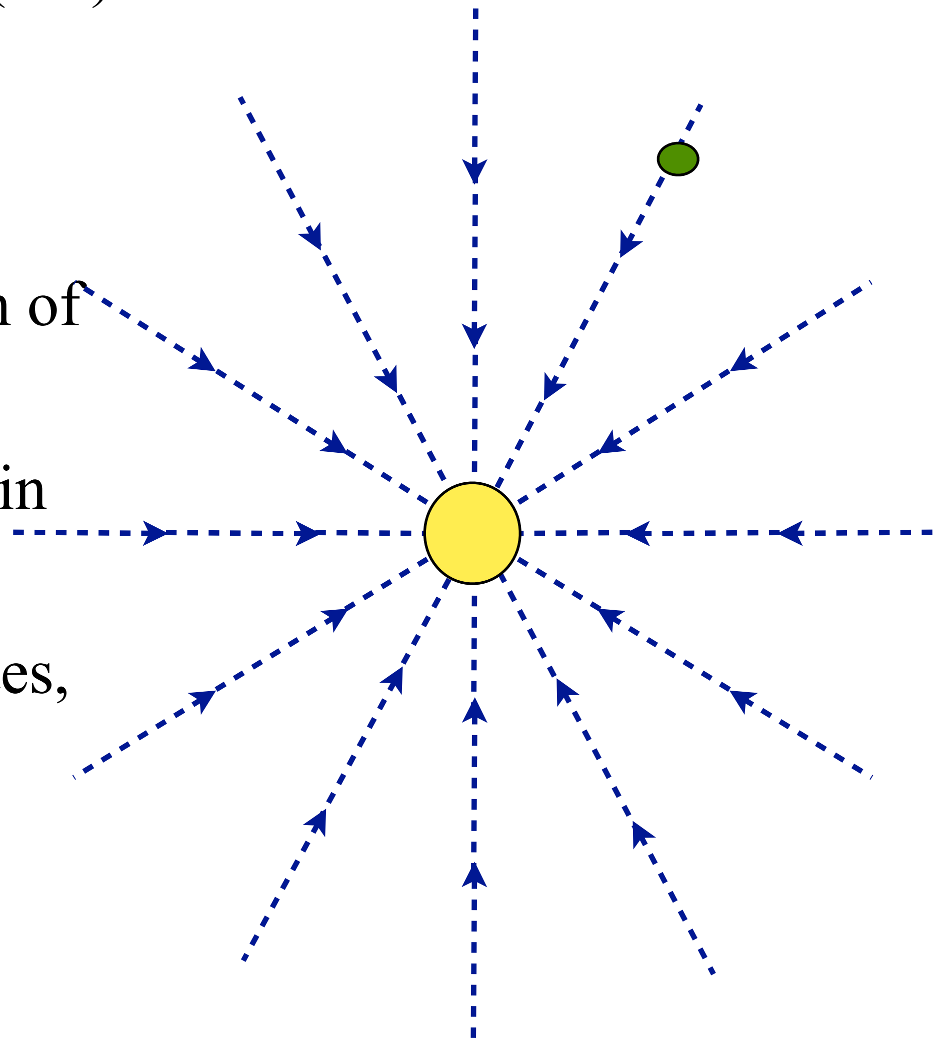
Luciano Maiani:
Lezione Fermi 10
Quantum Electro Dynamics, QED. Basic

1. Fields, waves and particles
2. Complex Fields and phases
3. Lowest order QED processes
4. Loops and divergent corrections
5. Two predictions of Dirac
6. The Shelter Island Conference, 1947
7. Como meeting, 1945.

1. Fields, waves ad particles

- Newton's action at a distance:
  - the Sun acts on the Earth (or proton on the electron), force directed from E to S, inversely proportional to the square of the distance
  - the Earth respondes to the instantaneous position of the Sun (???)

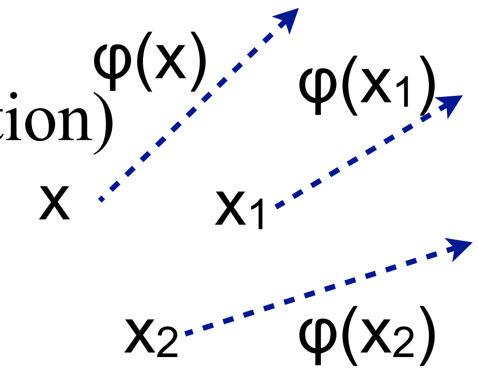
- A better picture:
  - Sun creates a "field"
  - a test body placed in the field in a given point feels the action of the field at that point
  - if the Sun moves, the new position is transmitted to the field in some time
  - not faster than the speed of light in vacuum: if Sun "evaporates, we feel the effect ony 8 minutes later



- This is "action at contact", ***hic et nunc***.
- Faraday and Maxwell develop the concept of field for electromagnetic phenomena where it is indispensable: there is nothing in the North to align a magnetic compass, but only a magnetic field at any point of the Earth!

waves and particles in the field

- Maxwell discovered that perturbations of the field propagate as waves
- in fact he finds: velocity of propagation= velocity of light
- these waves ***are*** light
- to describe propagation we have to assume that the field at one point (x) is coupled to the field in a point nearby (x_1): the Action (i.e. the Lagrange function) must contain terms which tend to equalize the field in x_1 to the field in x.



  - Terms of the form: $[\varphi(x)-\varphi(x_1)]^2$
  - to make them non vanishing when x_1 is very close to x, we divide by the distance (there are four directions in space time in which we could take the difference, so four derivatives, μ=0, 1, 2, 3, time is 0, sum over repated indices):

$$\frac{\phi(x_1)-\phi(x)}{d} \rightarrow \frac{\partial\phi(x)}{\partial x_\mu} = \partial_\mu\phi(x);\ [\frac{\phi(x_1)-\phi(x)}{d}]^2 \rightarrow \partial^\mu\phi(x)\partial_\mu\phi(x)$$

- if we move φ(x), e.g. by an oscillating charge, $\varphi(x_1)$ will follow, with some delay, ***in order to minimize the Action***;
- $\varphi(x_2)$ will follow $\varphi(x_1)$,...and so on:
- the disturbance propagates in space and time, with a velocity determined by the strenght of the coupling of the fields in nearby points.
- If the field is a quantum field, the state of the field is described by discrete Planck-Einstein quanta, i.e. particles of definite mass which propagate with some momentum p and energy E, with: $E^2 - p^2 = m^2$

2. Complex field and phases

- We have considered complex fields to describe charged particles
- take a field represented by a single complex quantity at each point, $\varphi(x) \neq \varphi^*(x)$
- observables will be real quantities, like e.g. $J=\varphi(x)\,\varphi^*(x)$, which is insensitive to the phase of $\varphi(x)$
- in formal language,
  - if we transform $\varphi(x) \to e^{i\alpha}\,\varphi(x)$, $\varphi^*(x) \to e^{-i\alpha}\,\varphi^*(x)$, then J is invariant: $J \to J$
  - this is true also if we choose the phase differently in different points: $\alpha(x) \neq \alpha(x_1)$
- The invariance under phase transformations of the observables which are taken in different points is desirable: why should the observer in Tokyo take the same phase choice as the one in Roma? Einstein would say: *no spooky action at a distance*!
- However, there are observables which depend upon fields in different points, like $[\varphi(x)-\varphi(x_1)]^*\,[\varphi(x)-\varphi(x_1)]$: what happens to them?

$$\frac{\phi(x_1)-\phi(x)}{d} \to \frac{e^{i\alpha(x_1)}\phi(x_1)-e^{i\alpha(x)}\phi(x)}{d} = e^{i\alpha(x)}\frac{\phi(x_1)+\phi(x)+i\Delta\alpha(x)\phi(x)}{d} =$$

$$= e^{i\alpha(x)}[\frac{\phi(x_1)-\phi(x)}{d} + i\frac{\Delta\alpha(x)}{d}\phi(x)]$$

A little simplification:

$$e^{i\alpha(x_1)} = e^{i\{[\alpha(x_1)-\alpha(x)]+\alpha(x)\}} =$$

$$\approx e^{i\alpha(x)}e^{i\Delta\alpha(x)} =$$

$$\approx e^{i\alpha(x)}[1+i\Delta\alpha(x)]$$

- there is an irregular term
- maybe adding another irregular term, the irregularities will cancel!??

An Abelian Gauge Theory

$$\frac{\phi(x_1)-\phi(x)}{d} \rightarrow e^{i\alpha(x)}[\frac{\phi(x_1)-\phi(x)}{d} + i\frac{\Delta\alpha(x)}{d}\phi(x)]$$

- using derivatives: $\partial_\mu\phi(x) \rightarrow e^{i\alpha(x)}[\partial_\mu\phi(x) + i\partial_\mu\alpha(x)\phi(x)]$
- let us consider a vector field (one vector $A_\mu(x)$ at each point x)
- and consider the transformation of: $\partial_\mu\phi(x) - iA_\mu(x)\phi(x)$

  under the combined transformations: $\phi(x) \rightarrow e^{i\alpha(x)}\phi(x);\ A_\mu(x) \rightarrow A_\mu + \partial\alpha(x)$
- we find:

$$\partial_\mu\phi(x) - iA_\mu(x)\phi(x) \rightarrow$$
$$\rightarrow e^{i\alpha(x)}[\partial_\mu\phi(x) + i\partial_\mu\alpha(x)\phi(x) - iA_\mu(x)\phi(x) - i\partial_\mu\alpha(x)\phi(x)] =$$
$$= e^{i\alpha(x)}[\partial_\mu\phi(x) - iA_\mu(x)\phi(x)] \quad \text{(!!!)}$$

- we can form invariant combinations with the "covariant derivative"

$$[\partial^\mu\phi^*(x) + iA^\mu(x)\phi^*(x)][\partial_\mu\phi(x) - iA_\mu(x)\phi(x)] \quad \text{(scalar field)}$$
$$\bar{\psi}(x)\gamma^\mu[\partial_\mu\psi(x) - iA_\mu(x)\psi(x)] \quad \text{(Dirac field)}$$

Electro Dynamics as a Gauge Theory

- It is known that the Electric and Magnetic fields of the Maxwell equations can be expressed in terms of a vector potential $A_\mu(x)$
- and that ***Electric and Magnetic fields remain unchanged if one adds a derivative to the vector potential: the transformation we have just introduced***
- We can identify the vector field and the transformations just introduced with the vector potential of Maxwell theory.
- The theory thus constructed with the covariant derivative is invariant under Abelian (phase) transformations, different in each space point.
- This is an Abelian Gauge Theory;
- the electron interacts with the vector field via the Action: e $A_\mu(x)J^\mu(x)$,

$$J_\mu = \bar{\psi}\gamma_\mu\psi$$

- e is a normalization constant identified with the electron charge.

The stories of QED

- Action: e $J_\mu A^\mu$
- electrons enter in the Action via the electromagnetic current: $$J_\mu = \bar{\psi}\gamma_\mu\psi$$
- electron and photon fields induce creation and annihilation of electrons/positrons/ photons, according to the scheme:

$$\bar{\psi} \approx [a^* e^{+ikx} + b e^{-ikx}];\ \psi \approx [a e^{-ikx} + b^* e^{+ikx}]$$
$$J \approx \bar{\psi}\psi : \text{initial} \to \text{final}$$
$$e^- \to e^-(a^* a);\ e^+ \to e^+(b b^*)$$
$$e^- e^+ \to 0(ba);\ 0 \to e^- e^+(a^* b^*)$$

$$A \approx [c e^{-ikx} + c^* e^{+ikx}] :$$
$$0 \to \gamma; \gamma \to 0$$

- the story of a process is made by lines of electrons/positrons going from one vertex to another, possibly starting from sources and ending to detectors;
- in each vertex a 3-body process takes places according to the table;
- photon lines also go from one vertex to another or in (fro sources) or out (to detectors)
- only connected diagrams matter (not made by two or more discnnected parts)
- each diagram (story) gives an amplitude, sum over stories, make the $|A|^2$ at the end, to get the probability.

3. Lowest order QED processes

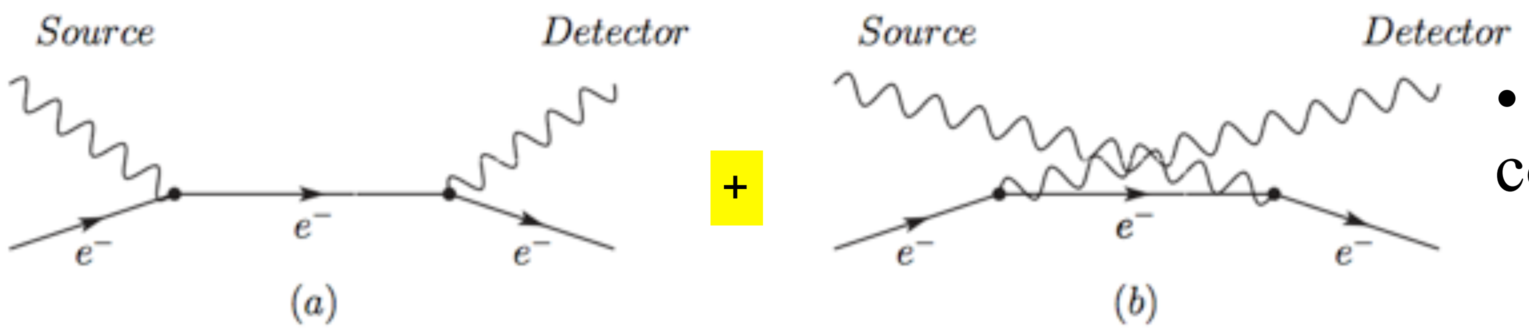
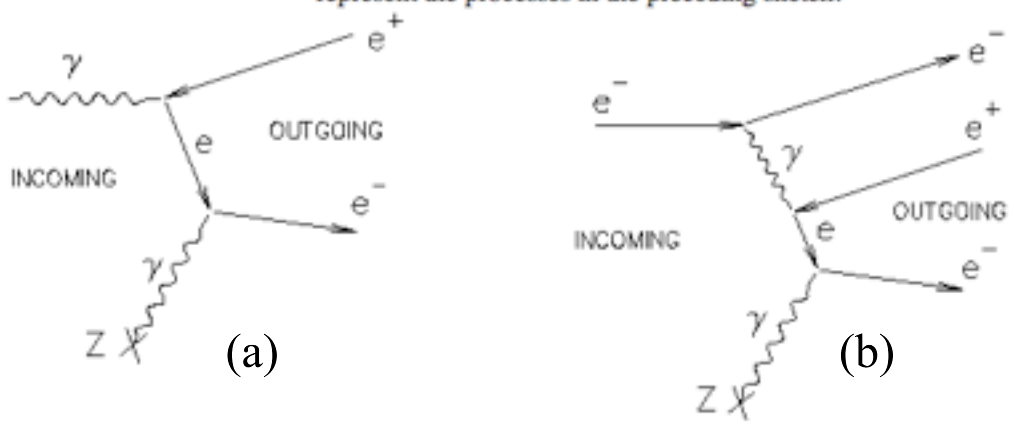
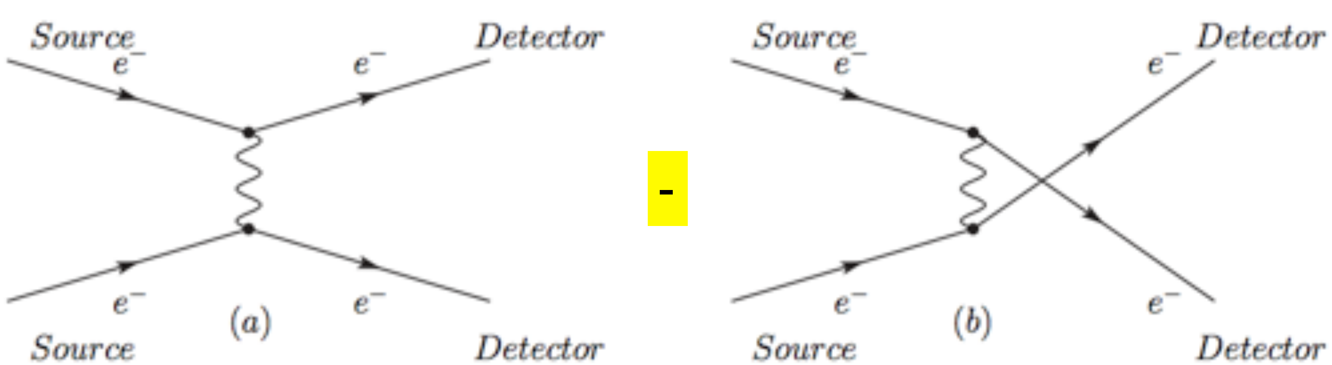
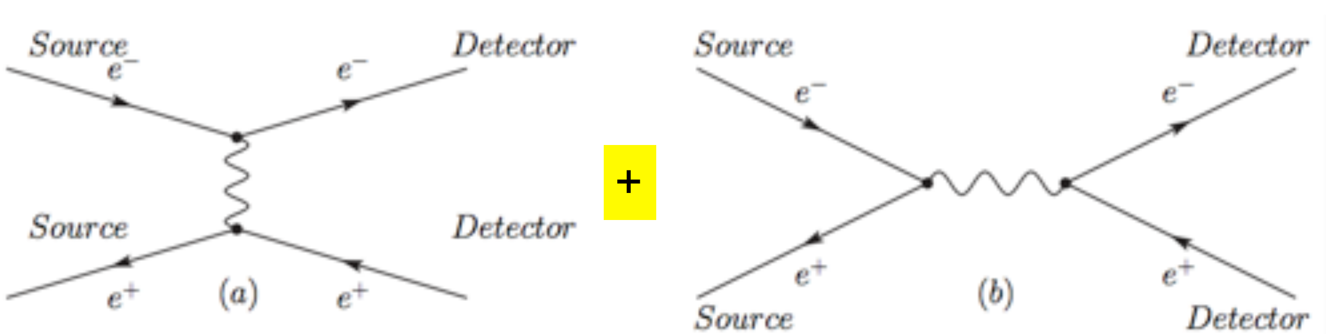


Figure: FEYNMAN DIAGRAMS for pair production by a gamma ray (left) or an electron (right). These represent the processes in the preceding sketch.







- Note: arrows follow the flow of the charge: positrons appear “as if” they were electrons running back in time

- Compton scattering (cross section computed by Klein and Nishina, 1928)

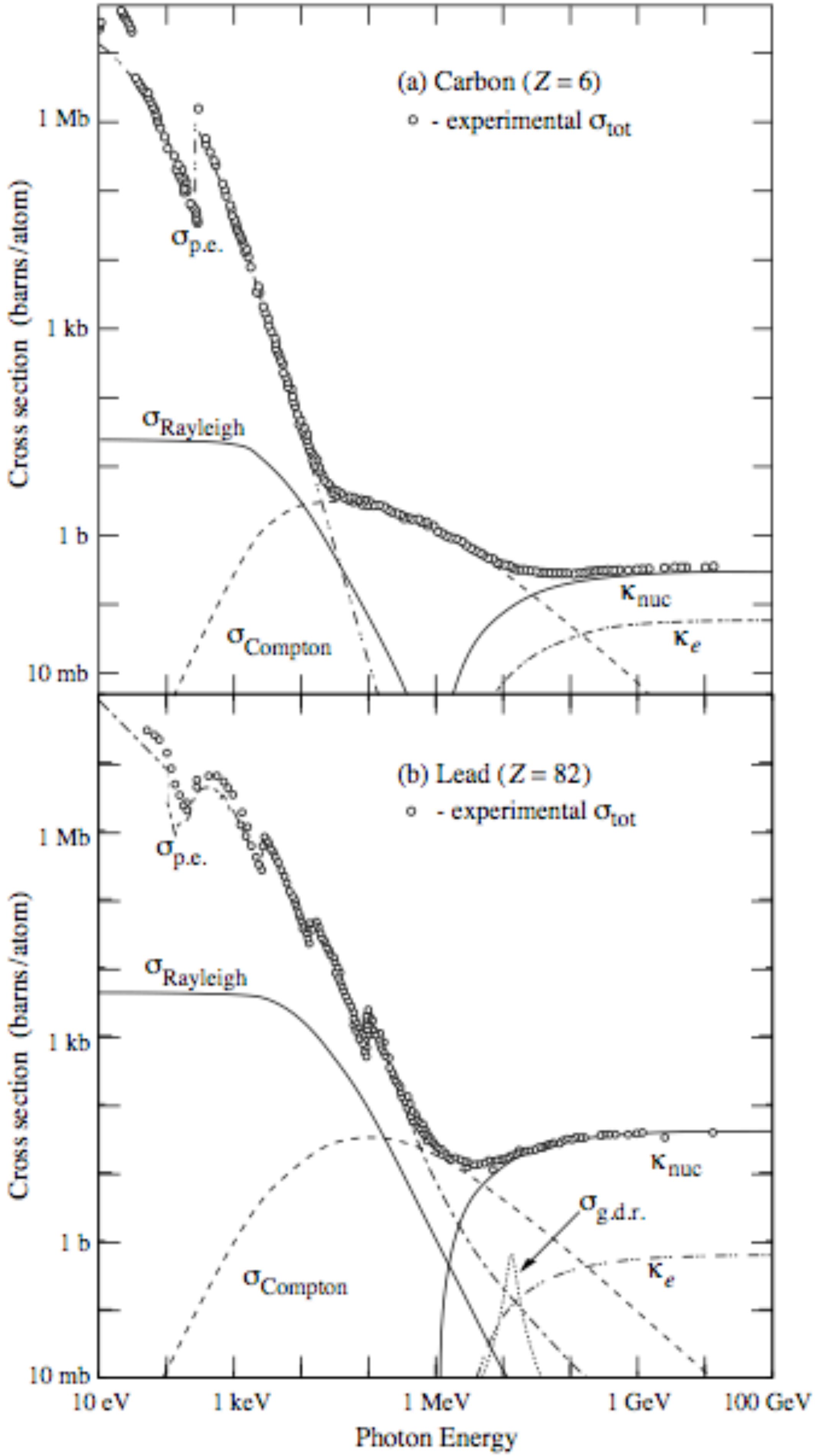
  same for e+

- Pair production on nuclei by photon (a) or electron (b)

- Moeller scattering (note the minus sign due to Fermi statistics)

  same for e+

- Bhaba scattering (1936)



Total photo-absorption in Carbon and Lead

J. Beringer et al. (Particle Data Group), Phys. Rev. D86, 010001 (2012). Experimental Methods and Colliders/ Passage of Particles through Matter

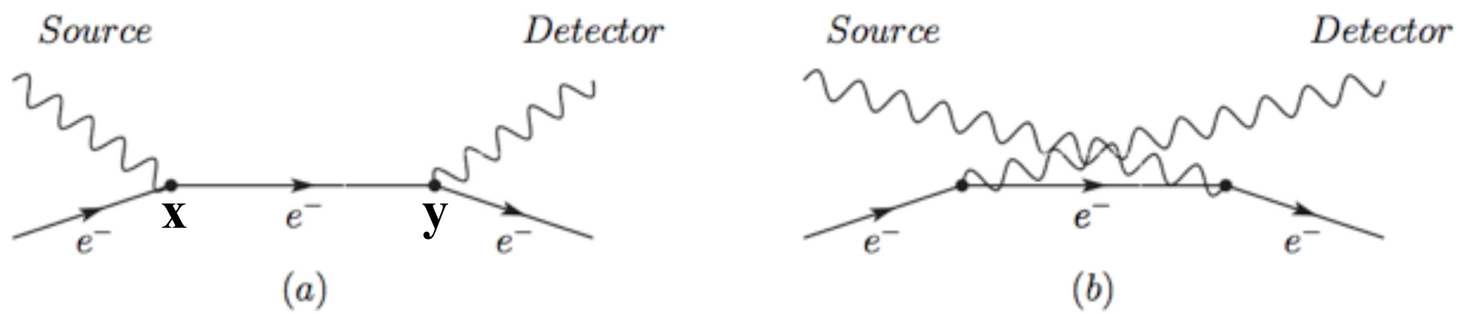
Figure 30.15: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes [48]:

$\sigma_{\text{p.e.}}$ = Atomic photoelectric effect (electron ejection, photon absorption)
σ_{Rayleigh} = Rayleigh (coherent) scattering–atom neither ionized nor excited
σ_{Compton} = Incoherent scattering (Compton scattering off an electron)
κ_{nuc} = Pair production, nuclear field
κ_e = Pair production, electron field
$\sigma_{\text{g.d.r.}}$ = Photonuclear interactions, most notably the Giant Dipole Resonance [49]. In these interactions, the target nucleus is broken up.

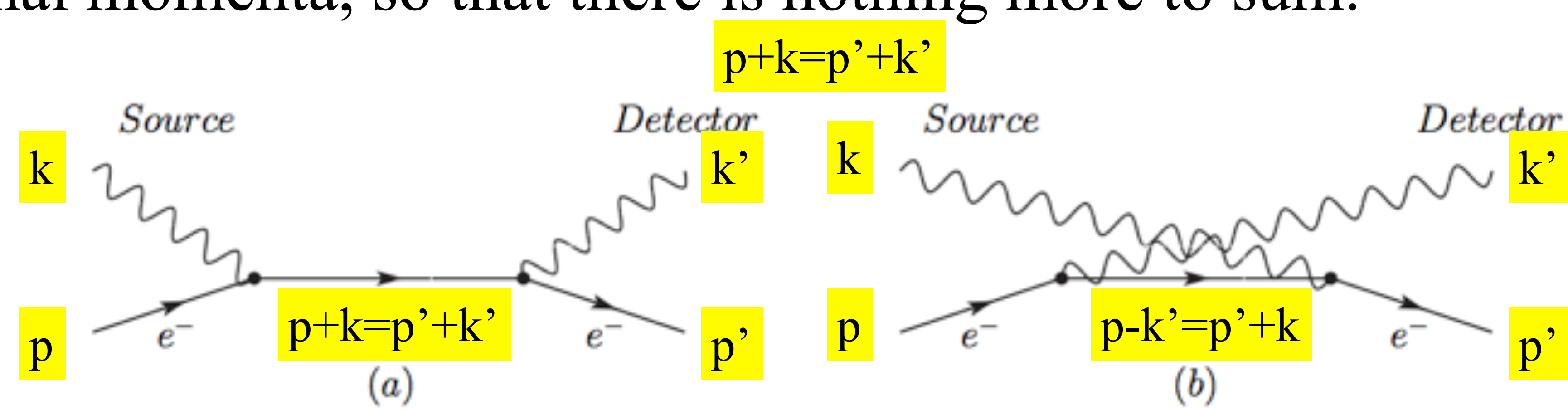
Original figures through the courtesy of John H. Hubbell (NIST).

- The control of photon absorption has many applications
- most important is the calculation of the dose absorbed by patients irradiated for cancer therapy, to minimize side harmful effects.

Tree Diagrams

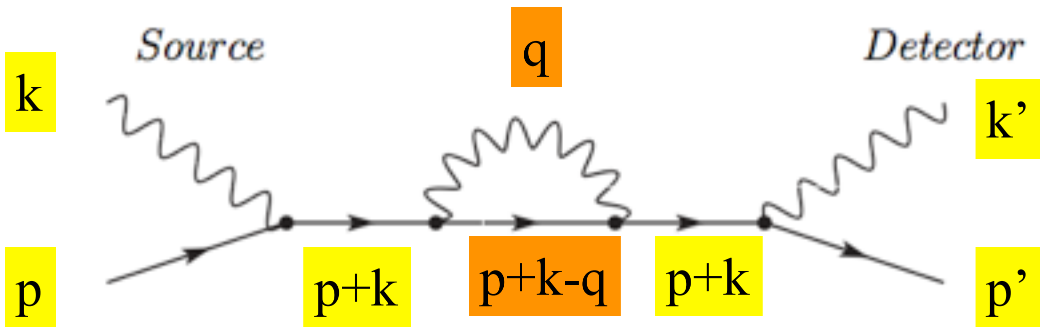


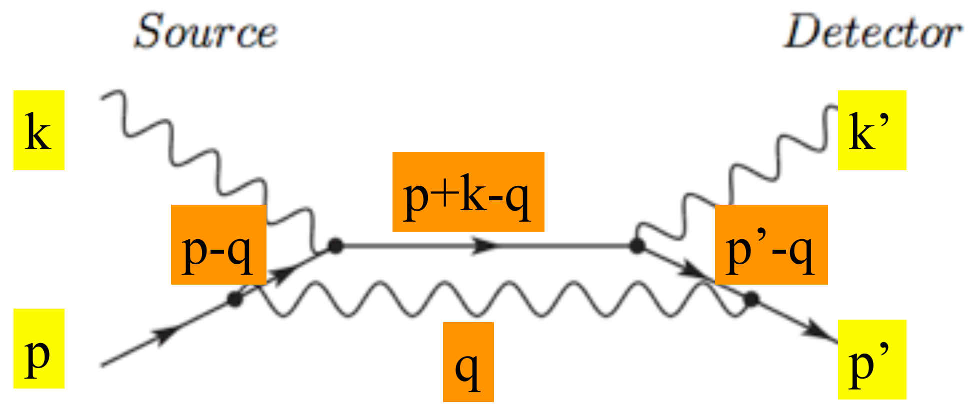
- tree diagram = no closed loops
- suppose we have incoming and outgoing particles with definite momenta (sources and detectors are very far..)
- according to instructions, we must sum the amplitudes corresponding to all values of the coordinates where interactions happen (vertices)
- summing over each coordinate implies that momentum is conserved at each vertex
- as a consequence, external momenta are conserved: Ptot,in = Ptot,out
- for a tree diagram, in addition, the external momenta fix the values of the internal momenta, so that there is nothing more to sum.



4. Diagrams with loops give (sometime) divergent results

- if we want to compute higher order effects to a given process, e.g. Compton scattering, we have keep fixed the external legs of the diagram and add new vertices and new internal lines, we encounter diagrams with ***loops***.





- Consider the first diagram, assign the initial momenta and proceed to determine the momenta of the internal lines by using momentum conservation
- we remain with one undetermined momentum in the loop, q, and we have to sum the amplitude, A(q), over all values of q, ***up to infinity***!!!
- there are cases where the sum gives a finite result, because A(q) goes enough quickly to zero for large q (the "box" diagram, see)
- but cases, such as the first in the figure, where the sum gives an ***infinite result***
- corrections to the lowest order, which by the way gives a good approximation to the experimental value, are infinite !!!! what happens? is QED sick? inconsistent?
- the problem appeared, in the '30s . At that time:
  - calculations were done in a very complicated theory (now we call it *old fashioned perturbation theory*)
  - there were no precise data to confront with
- Dirac thought that drastic changes were needed to make a consistent theory, perhaps as drastic as the passage from the orbit theory of Bohr to quantum mechanics.

5. Two predictions of the Dirac equation
I. the electron anomaly

- An electron on a closed orbit generates a magnetic field (H. A. Rowland's experiment, 1876) as if it were magnetic dipole
- As Bohr indicated, the natural unit for the magnetic moment associated to an orbital angular momentum L, (= x∧p), is the "Bohr magneton", μ_B

$$\mu_B = \frac{e\hbar}{2mc}$$

- in a magnetic field H along z, an atom with one electron in orbit L has an additional energy $E=\mu_B\, g_L L_z\, H (= E_L)$.
- We have introduced the additional factor g_L, the Lande' factor, however we know with Bohr that $g_L=1$, as confirmed by measurements of the *normal Zeeman effect;*
- the spin of the electron was introduced by Uhlenbeck an Goudsmith to explain the *anomalous Zeeman effect*, by an additional magnetic interaction term, $E_s=\mu_B\, g_s s_z H$, so that $E=E_L+E_s=\mu_B\, H(g_s s_z + L_z)$
- spectroscopic data suggested $g_s \sim 2$.
- In 1928, $g_s=2$ was derived directly from Dirac's equation: a great success!!
- since then, for an ***elementary spin 1/2 particle*** we define the magnetic anomaly as the deviation from Dirac's value:

$$a = \frac{g-2}{2},\ a_e = 0 \text{ predicted by Dirac's equation}$$

- proton and neutron are certainly not elementary and have large anomalies:

$$g_p = 2.792847356 \pm 0.000000023 \neq 2,\ \ g_n = 1.9130427 \pm 0.0000005 \neq 0$$

II. The $2P_{1/2}$ and $2S_{1/2}$ degeneracy

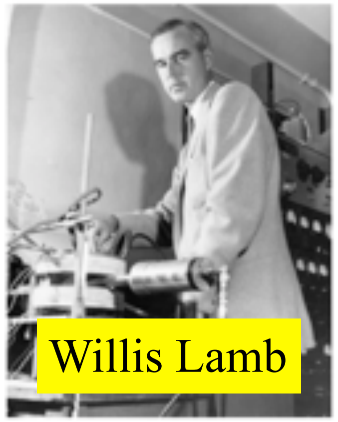
- The Bohr formula for the hydrogen atom gives the energy in terms of one integer quantum number, n (cfr. Lezione 3) $$E_n = -\frac{e^2}{2n^2 R_B} = -\frac{1}{2n^2}\alpha^2 mc^2 \approx -\frac{13.3}{n^2}\ \mathrm{eV}$$
- a single value n corresponds to states with different orbital momentum, L=0, 1,..., n-1, all with the same energy; states with L=0 and 1 are denoted with the symbols S and P.
- The Dirac equation introduces the electron spin and removes partially this degeneracy
- the result is that, for given n, the energy depends only on the total angular momentum, which may be equal to L +1/2 or L-1/2 (only +1/2 for L=0).
- The states with n=2 may have L=0 (S) , J=1/2, or L=1 (P), J=1/2 and 3/2
- thus the states $2S_{1/2}$ and $2P_{1/2}$ have the same n=2 and the same J=1/2, therefore are predicted to be degenerate
- spectroscopic measurements agreed in the ‘30s with equal energy, within errors of percent.

The Shelter Island Conference, 1947

Discussing physics informally (left to right): R. Feynman, H. Feshbach, J. Schwinger. *Reprinted from Shelter Island II*

The rules of QED had been known since the early days of quantum mechanics, having been formulated by Fermi, Dirac and others. But it was also known that as soon as one went beyond the lowest order of perturbation theory, the predictions ceased to make sense, seemingly giving divergent answers for physical quantities. Physicists had wrestled with these issues during the thirties, and had largely put them aside during the wartime period. At the Shelter Island Conference, 24 scientists, most of them theoretical physicists, gathered to take serious stock of the situation.

Julian Schwinger: "It was the first time that people who had all this physics pent-up in them for five years could talk to each other without somebody peering over their shoulders and saying, 'Is this cleared?'



Willis Lamb

Polykarp Kusch

The birth of modern QED

- Willis Lamb reported a measurement of the energy difference of $2S_{1/2}$-$2P_{1/2}$ since then known as the *Lamb shift*, corresponding to: ν=1057 MHz, i.e. 4.372 10^{-6} eV.
- Isidor Rabi reported a precise measurement of the magnetic moment of the electron by P. Kusch and H. M. Foley: a_e=0.00114(8) (in parenthesis the error on the last digit)
- R.P. Feynman reported on his new version of QED, the Feynman diagrams
- on the train back from Shelter Island, H. Bethe derived the first calculation of the Lamb shift. Shortly after, Schwinger computed $a_e=\alpha/(2\,\pi)\simeq 0.00116$.

Edoardo Amaldi, 1978
The years of reconstruction: first post-war meeting, Como 1945

A few months after the liberation of Milan, Polvani organized at Como a Physics Conference for celebrating the second centenary of Alessandro Volta's birth. From Rome A. Giacomini, director of the Istituto di Elettroacustica O.M. Corbino, and I took part in the meeting.

We reached our destination after a 36 hours trip with our rucksacks full of supplies, having crossed on foot pontoon bridges over a few rivers, on the banks of which the trains mainly composed of cattle carriages stopped. This was the first reunion of the physicists of Central and South Italy with those of North Italy. Besides Polvani, Persico, Perucca, Carrelli and Somigliana there were Rostagni, Caldirola and a few younger physicists including Carlo Salvetti and Giorgio Salvini, who had been formed during the difficult war years by studying when on leave from their military duties, the first under the influence of Giovanni Gentile jr., the other in the wake of Giuseppe Cocconi who had been mobilized at the beginning of 1941 and moved, a few months later to Rome, as I will say below.