A DIFFERENT DERIVATION OF THE VERLET ALGO.

$$m \frac{dv}{dt} = F(r(t))$$

We integrate this equation in $[t, t+\Delta t]$

$$m \int_t^{t+\Delta t} dt \frac{dv}{dt} = \int_t^{t+\Delta t} F(r(t))\, dt$$

$$v(t+\Delta t) - v(t) = \frac{1}{m} \int_t^{t+\Delta t} F(r(t))dt$$

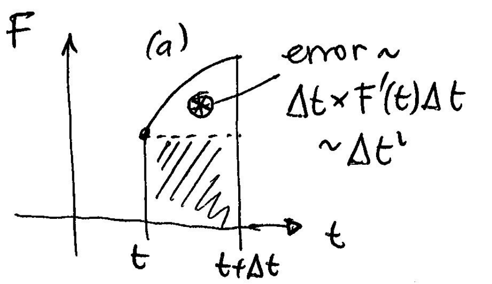
Up to here : EXACT

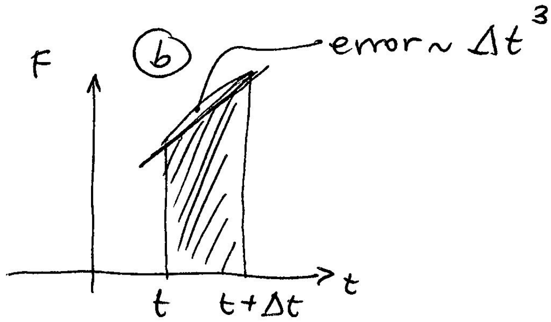
We approximate the integral :

(a) $\displaystyle\int_t^{t+\Delta t} F(r(t))\, dt = F(r(t))\Delta t + O(\Delta t^2)$

note the difference in corrections

(b) $\displaystyle\int_t^{t+\Delta t} F(r(t))dt = \frac{\Delta t}{2}\Big(F(r(t+\Delta t)) + F(r(t))\Big) + O(\Delta t^3)$





<u>VERLET</u> corresponds to b

$$v(t+\Delta t) = v(t) + \frac{\Delta t}{2m}\left(F(r(t)) + F(r(t+\Delta t)\right) + O(\Delta t^3)$$

The second equation can be obtained as

$$\frac{dx}{dt} = v(t)$$

$$x(t+\Delta t) - x(t) = \int_t^{t+\Delta t} dt\, v(t) = \frac{\Delta t}{2}\left(v(t+\Delta t)+v(t)\right) + O(\Delta t^3)$$

$$= \frac{\Delta t}{2}\left[v(t) + \frac{\Delta t}{2m}\left(F(t+\Delta t)+F(t)\right) + v(t)\right] + O(\Delta t^3)$$

$$= v(t)\,\Delta t + \frac{\Delta t^2}{4m}\left(F(t+\Delta t)+F(t)\right) + O(\Delta t^3)$$

$$\approx v(t)\,\Delta t + \frac{\Delta t^2}{2m} F(t) + O(\Delta t^3)$$