MOMENTUM CONSERVATION

Consider a system in the <u>ABSENCE</u> of EXTERNAL FORCES

All forces are internal and appears in pairs (3° principle of Newtonian dynamics)

<u>Obvious</u> if $U(r_1 \dots r_n) = \sum_{i<j} V(|r_i - r_j|)$

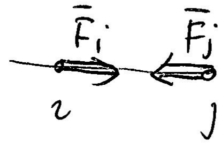
The term $V(|r_i - r_j|)$ gives rise to a force acting on particles i and j

$$\text{on } i \rightarrow \bar{F}_i = -\frac{\partial}{\partial \bar{r}_i} V(|r_i - r_j|) = -\frac{\partial r_{ij}}{\partial \bar{r}_i} \frac{\partial V(r_{ij})}{\partial r_{ij}} \qquad r_{ij} = |\bar{r}_i - \bar{r}_j| = |r_j - r_i|$$

$$= -\frac{(\bar{r}_i - \bar{r}_j)}{|r_i - r_j|} \frac{\partial V(r_{ij})}{\partial r_{ij}}$$

$$\text{on } j \rightarrow \bar{F}_j = -\frac{\partial}{\partial \bar{r}_j} V(|r_i - r_j|) = -\frac{(\bar{r}_j - \bar{r}_i)}{|r_i - r_j|} \frac{\partial V(r_{ij})}{\partial r_{ij}}$$

$$\bar{F}_i + \bar{F}_j = 0$$



Verlet - velocity for particle i of mass m_i

$$\bar{v}_i(t+\Delta t) = v_i(t) + \frac{1}{2m_i}\left(F_i(t+\Delta t) + F_i(t)\right)$$

$$P_i(t+\Delta t) = \sum_i m_i v(t+\Delta t) =$$

$$= \sum_i m_i \bar{v}(t) + \frac{1}{2}\sum_i \left(F_i(t+\Delta t) + F_i(t)\right)$$

0 (forces cancel in pairs)

$$= P(t)$$

$$\boxed{P = \text{is conserved}}$$

CORRECTION:
P(t+Delta t), there's no index i; i should be added to the velocities v