ENERGY FLUCTUATIONS AND SPECIFIC HEAT

[CANONICAL ENSEMBLE]

Definitions $E = \langle H \rangle$

$C_V = \frac{dE}{dT}$ specific heat at constant volume V

$$\langle H \rangle = \frac{1}{Z} \int [dpdq]\, H e^{-\beta H} = -\frac{1}{Z} \frac{\partial}{\partial \beta} \int [dpdq]\, e^{-\beta H}$$

$$= -\frac{1}{Z} \frac{\partial Z}{\partial \beta} = -\frac{\partial \ln Z}{\partial \beta} = \frac{\partial (\beta F)}{\partial \beta}$$

$$C_V = \frac{\partial}{\partial T} \langle H \rangle = -\frac{1}{kT^2} \frac{\partial}{\partial \beta} \left[\frac{\int [dpdq]\, H e^{-\beta H}}{\int [dpdq]\, e^{-\beta H}} \right]$$

$$= -\frac{1}{kT^2} \left[-\frac{\int [dpdq\, H^2 e^{-\beta H}}{\int [dpdq]\, e^{-\beta H}} + \frac{\left(\int [dpdq\, H e^{-\beta H} \right)^2}{\left(\int dpdq\, e^{-\beta H} \right)^2} \right]$$

$$= \frac{1}{kT^2} \left[\langle H^2 \rangle - \langle H \rangle^2 \right]$$

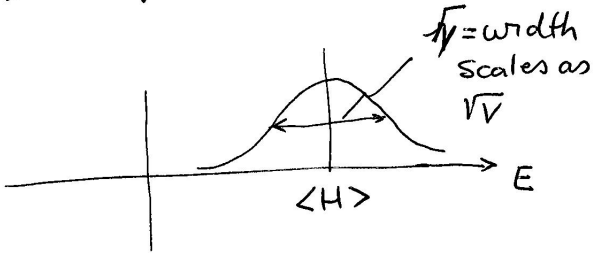
The specific heat

$$\langle H^2 \rangle - \langle H \rangle^2 = kT^2 C_V(T)$$

The energy is extensive $\Rightarrow$ C_V is extensive

$E \sim V, N$ $\quad C_V \sim V, N$

$$\langle H^2 \rangle - \langle H \rangle^2 \sim V, N$$



$\frac{E}{N} = \frac{E}{V} \cdot \frac{1}{\rho}$

↑ energy per particle