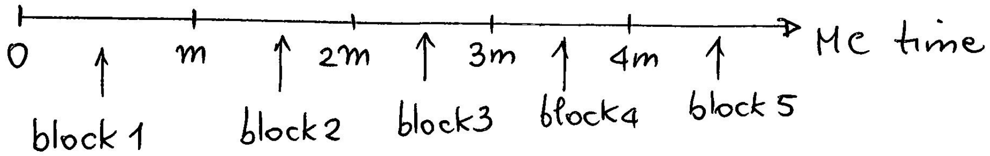
IMPROVED BLOCKING METHOD

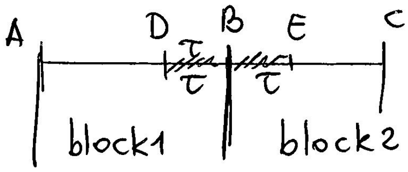
The blocking method converges slowly. To obtain reasonable estimates one needs blocks of length m with $m \gg 10\tau$.

The reason can be understood as follows



The method essentially assumes that successive blocks are UNCORRELATED

This is true only far from the boundaries



The data in DB that belong to block 1 are correlated with the data in BE that belong to block 2

The length of DB and DE is of order τ.

The boundaries are negligible only if $m \gg \tau$.
THIS EXPLAINS the slow convergence.

The bulk of the corrections is due to the correlations between successive blocks.

The idea of the IMPROVED BLOCKING METHOD is that of including these correlations

THE PRACTICAL PROCEDURE

- Compute $F_i^{(1)}$ $i: 0, \dots \frac{N}{2}-1$ $\boxed{\frac{N}{2}=m}$

$$\mathrm{Var}\, F^{(1)} = \frac{1}{m}\sum_{i=0}^{m-1}\left(F_i^{(1)}\right)^2 - \left(\frac{1}{m}\sum_{i=0}^{m-1} F_i^{(1)}\right)^2$$

$$C_1^{(1)} = \frac{1}{m-1}\sum_{i=0}^{m-2} F_i^{(1)} F_{i+1}^{(1)} - \left(\frac{1}{m}\sum_{i=0}^{m-1} F_i^{(1)}\right)^2$$

$C_1^{(1)}$ is the autocorrelation function of $F_i^{(1)}$ at time difference 1 (the autocorrelation between successive blocks)

Then we get

$$\sigma_1^2 = \frac{1}{m}\left[\mathrm{Var}\, F^{(1)} + 2C_1^{(1)}\right]$$

- We repeat the same procedure for $F_i^{(2)}$ (there are $m = \frac{N}{4}$ data)

AGAIN THE EXAMPLE $C_f(k) = e^{-k/10}$ $\tau = 10$

k	$N\sigma_k^2$ (std)	$N\sigma_k^2$ (imp)
2	3.55	8.98
3	6.26	13.84
4	10.05	18.00
5	14.03	19.77
6	16.90	20.01
	↓	↓
∞	20.00	20.00

With the improved method $k \approx 4-5$ is enough