AN ALGORITHM TO GENERATE GAUSSIAN RANDOM NUMBERS

We wish to generate numbers X_i such that

$$\text{Prob}(x < X_i < x + dx) = f(x)\,dx$$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

GAUSSIAN RANDOM NUMBERS with

0 average : $\langle x \rangle = 0$

1 variance ; $\langle x^2 \rangle = 1$

The algorithm generates PAIRS of random numbers.

$$\text{Prob}(x < X_1 < x + dx \;\&\; y < X_2 < y + dy) = f(x)\,f(y)\,dx\,dy$$

$$= \frac{1}{2\pi} e^{-(x^2+y^2)/2}\,dx\,dy$$

The IDEA :

CHANGE VARIABLES SO THAT IN THE NEW variables the distribution IS UNIFORM so that we use the RNG RAN( ).

The probability density for the pair is

$$\pi = \frac{1}{2\pi} e^{-(x^2+y^2)/2} \, dx \, dy$$

Define $\begin{cases} x = R\cos\theta \\ y = R\sin\theta \end{cases}$ (polar coordinates) $0 \le R \le \infty$, $0 \le \theta \le 2\pi$

$$\pi = \frac{1}{2\pi} e^{-R^2/2} \, R \, dR \, d\theta$$

Note $\quad d e^{-R^2/2} = -R e^{-R^2/2} \, dR$

The sign is irrelevant: when one changes variables one should take the ABSOLUTE VALUE of the jacobian

Define $\quad Z = e^{-R^2/2} \qquad 0 \le Z \le 1 \qquad R = \sqrt{-2 \ln Z}$

$$\pi = \frac{1}{2\pi} \, dZ \, d\theta \leftarrow$$ UNIFORM PROBABILITIES in $[0,1]$ (Z) and $[0, 2\pi]$ (θ)

ALGORITHM TO GENERATE X_1 and X_2

$Z = RAN(\ )$

$\theta = (2\pi) * RAN(\ )$

$R = \sqrt{-2 * \ln Z}$

$$\left.\begin{aligned} X_1 &= R * \cos\theta \\ X_2 &= R * \sin\theta \end{aligned}\right\}$$ A PAIR OF GAUSSIAN RANDOM NUMBERS