## Chemical equilibrium



A dynamic equilibrium: the speed in one direction is balanced by the speed in the other. Although no macroscopic change is visible, action is taking place.

# Difference between subscript in a chemical formula and a stoichiometric coefficient. 

$\mathrm{H}_{2} \mathrm{O}$ one molecule of water
2 atoms of H and 1 atom of O
$2 \mathrm{H}_{2} \mathrm{O}$ two molecules of water
4 atoms of H and 2 atoms of O
$\mathrm{H}_{2} \mathrm{O}_{2}$ hydrogen peroxide
2 atoms of H and 2 atoms of O

## STOICHIOMETRY OF CHEMICAL REACTIONS

Study of quantitative relationships among compounds that undergo a chemical transformation

Law of mass conservation (Lavoisier)
In a chemical reaction atoms are neither created nor destroyed.

## Balancing of chemical reactions


balanced raction
a) $\mathrm{CH}_{4}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}+\mathrm{H}_{2} \mathrm{O}$ not balanced reaction
b) $\mathrm{CH}_{4}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}$ not balanced reaction
c) $\mathrm{CH}_{4}+2 \mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}$ balanced reaction

Combustion of octane produces carbon dioxide and water

$$
\begin{array}{ll}
\mathrm{C}_{8} \mathrm{H}_{18}+\mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}+\mathrm{H}_{2} \mathrm{O} & \text { not balanced raction } \\
\mathrm{C}_{8} \mathrm{H}_{18}+\mathrm{O}_{2} \rightarrow 8 \mathrm{CO}_{2}+9 \mathrm{H}_{2} \mathrm{O} & \begin{array}{l}
\text { balanced for } \mathrm{C} \text { and } \mathrm{H} \\
\text { O atoms are not balanced }
\end{array} \\
\mathrm{C}_{8} \mathrm{H}_{18}+12.5 \mathrm{O}_{2} \rightarrow 8 \mathrm{CO}_{2}+9 \mathrm{H}_{2} \mathrm{O} & \begin{array}{l}
\text { all atoms are balanced, but } \\
\text { coefficients are not integers }
\end{array}
\end{array}
$$

$$
2 \mathrm{C}_{8} \mathrm{H}_{18}+25 \mathrm{O}_{2} \rightarrow 16 \mathrm{CO}_{2}+18 \mathrm{H}_{2} \mathrm{O}
$$

NB: atom numbers are balanced, mass is conserved.

Chemical equations can indicate the physical state of reagents and products.

| Gas | $(g)$ |
| :--- | :---: |
| Liquid | $(1)$ |
| Solid | $(s)$ |
| aqueous solution | $(a q)$ |

$$
2 \mathrm{C}_{8} \mathrm{H}_{18}(\mathrm{I})+25 \mathrm{O}_{2}(\mathrm{~g}) \rightarrow 16 \mathrm{CO}_{2}(\mathrm{~g})+18 \mathrm{H}_{2} \mathrm{O}(\mathrm{I})
$$

$$
\begin{aligned}
& \mathrm{Na}(s)+\mathrm{H}_{2} \mathrm{O}(\prime) \rightarrow \mathrm{NaOH}(a q)+\mathrm{H}_{2}(g) \\
& \mathrm{Na}(s)+2 \mathrm{H}_{2} \mathrm{O}(\Omega) \rightarrow \mathrm{NaOH}(a q)+\mathrm{H}_{2}(g) \\
& \mathrm{Na}(s)+2 \mathrm{H}_{2} \mathrm{O}\left(\Omega \rightarrow 2 \mathrm{NaOH}(a q)+\mathrm{H}_{2}(g)\right. \\
& 2 \mathrm{Na}(s)+2 \mathrm{H}_{2} \mathrm{O}(\Omega) \rightarrow 2 \mathrm{NaOH}(a q)+\mathrm{H}_{2}(g)
\end{aligned}
$$

Difference between EQUILIBRIUM and STEADY STATE
Gas burner

$$
\mathrm{CH}_{4}+2 \mathrm{O}_{2} \rightarrow \mathrm{CO}_{2}+2 \mathrm{H}_{2} \mathrm{O}
$$

VAPOUR PRESSURE


Isolated system


Open system

## CHEMICAL EQUILIBRIUM

Chemical equilibrium is the dynamic state of a system, characterized by the formation of products and reagents at the same velocity.

$$
\mathrm{CH}_{3} \mathrm{COOH}+\mathrm{H}_{2} \mathrm{O} \rightleftharpoons \mathrm{CH}_{3} \mathrm{COO}^{-}+\mathrm{H}_{3} \mathrm{O}^{+}
$$



## REVERSIBILITY

Chemical reactions are reversible processes. There are conditions of concentration, temperature and pressure in which reactants and products coexist at equilibrium.
a) $2 \mathrm{NH}_{3} \rightarrow \mathrm{~N}_{2}+3 \mathrm{H}_{2}$
b) $\mathrm{N}_{2}+3 \mathrm{H}_{2} \rightarrow 2 \mathrm{NH}_{3}$

$$
2 \mathrm{NH}_{3} \rightleftarrows \mathrm{~N}_{2}+3 \mathrm{H}_{2}
$$

$2: 1: 3$


$\checkmark$ Dynamic equilibrium ( $\rightarrow=\leftarrow$ )
$\checkmark$ Same equlibrium, starting from reactants or products
$\checkmark$ It corresponds to an energy minimum

## The equilibrium constant

At equilibrium there is a correlation between the concentrations of reactants and products.

In the reaction between iodine and hydrogen:

$$
\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g}) \rightleftarrows 2 \mathrm{HI}(\mathrm{~g})
$$

Experiments have shown that the ratio between the concentration of HI and the product of $\mathrm{H}_{2}$ ed $\mathrm{I}_{2}$, is constant if temperature is constant.

$$
\frac{[\mathrm{HI}]^{2}}{\left[\mathrm{H}_{2}\right] \cdot\left[\mathrm{I}_{2}\right]}=\operatorname{costant}(\mathrm{K}) \text { at equilibrium }
$$

NB: molar concentrations are indicated by [ ], therefore:
$[\mathrm{HI}]=$ molar concentration of hdrogen iodide
$\left[\mathrm{H}_{2}\right]=$ molar concentration of molecular hydrogen, etc.

If the concentrations of $\mathrm{H}_{2}$ and $\mathrm{I}_{2}$ in a container are initially $0.0175 \mathrm{~mol} / \mathrm{L}$ at $425^{\circ} \mathrm{C}$ and HI is absent.


If $\mathrm{H}_{2}$ ed $\mathrm{I}_{2}$ are not present and [HI] is $0.04 \mathrm{~mol} / \mathrm{L}$ a $425^{\circ} \mathrm{C}$.


The initial amounts of reagents and products is irrelevant, at equilibrium their ratio will be the same.

## EQUILIBRIUM CONSTANT AND THE LAW OF MASS ACTION

Since at equilibrium the concentrations of the components are constant, their ratio will be a constant, this is the law of mass action.


The constant is called the equilibrium constant $K_{c}$. It is a thermodynamic constant that depends only on concentrations and on temperature, the " c " indicates that it is expressed in concentrations.


Law of mass action
$\mathrm{K}_{c}$ will be expressed in units that depend on the algebraic sum of the exponents or that can be dimensionless if $(a+b)=(c+d)$.
(The latter condition means that there is no change in the number of moles during the reaction)

## Equilibium constant

For a generic chemical raction at a certain temperature

$$
a A+b B \rightleftarrows c C+d D
$$

We can define an equilibrium constant

$$
\mathrm{K}=\frac{[\mathrm{C}]^{\mathrm{c}} \cdot[\mathrm{D}]^{\mathrm{d}}}{[\mathrm{~A}]^{\mathrm{a}} \cdot[\mathrm{~B}]^{\mathrm{b}}}
$$

In the equilibrium constant:

- All concentrations are the ones at equilibrium.
- Concentrations of the products appear in the numerator of $K$ and those of reactants in the denominator.
- Each concentration is raised to a power equal to the stoichiometric coefficient of the corresponding species in the balanced reaction.
- Only species in the same physical phase appear in the expression of K.
- The dimension of $K$ depends on the reaction


## $2 \mathrm{NH}_{3} \rightleftarrows \mathrm{~N}_{2}+3 \mathrm{H}_{2}$ <br> $\mathrm{K}_{\mathrm{c}}=\left[\mathrm{N}_{2}\right]\left[\mathrm{H}_{2}\right]^{3} /\left[\mathrm{NH}_{3}\right]^{2}$

The magnitude of $\mathrm{K}_{\mathrm{c}}$ indicates whether the reaction is shifted toward the formation of products or reagents.

Knowing $K_{c}$ one can determine the relative amounts of reactants and products present at equilibrium.

The $K_{c}$, while representing the same reaction, can have different values depending on how the reaction is written.
It is therefore very important to know "how" the reaction is written.

$$
\begin{array}{ll}
\mathrm{H}_{2}+\mathrm{I}_{2} \rightarrow 2 \mathrm{HI} & \mathrm{~K}_{\mathrm{c}}=[\mathrm{HI}]^{2} /\left[\mathrm{H}_{2}\right]\left[\mathrm{I}_{2}\right] \\
2 \mathrm{HI} \rightarrow \mathrm{H}_{2}+\mathrm{I}_{2} & \mathrm{~K}_{\mathrm{c}}{ }_{c}=\left[\mathrm{H}_{2}\right]\left[\mathrm{I}_{2}\right] /[\mathrm{HI}]^{2}
\end{array}
$$

The equilibrium constant of a reaction written in one direction and that of the same reaction written in the opposite direction have reciprocal values.

$$
K_{c}=1 / K^{\prime}{ }_{c}
$$

## HETEROGENEOUS EQUILIBRIUM

In equilibria in homogeneous systems all the components must be taken into account, while in heterogeneous systems we consider that the components in the condensed phase (solid or liquid) have a constant "concentration" (not "null" or equal to 1!), therefore their concentration can be incorporated into the equlibrium $K$.

Equilibrium
$\mathrm{CaCO}_{3}$ (s) $\rightleftarrows \mathrm{CaO}{ }_{\text {(s) }}+\mathrm{CO}_{2}$ (g)
Constant $K_{p}$ is $\quad K_{p}=P_{\text {co2 }}$
Constant $K_{c}$ is $\quad K_{c}=\left[\mathrm{CO}_{2}\right]$

$\mathrm{T}=800^{\circ} \mathrm{C}, \mathrm{P}=190 \mathrm{mmHg}$

Reactions involving solids (heterogeneous)

The concentration of a solid is determined by its density, and density is constant.


In general, the concentration of a solid (reactant or product) in a reaction does not appear in the expression of the equilibrium constant.

Reactions in aqueous solution

$$
\begin{aligned}
& \text { Reactions in aqueous solution } \\
& \mathrm{NH}_{3}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{I})=\mathrm{NH}_{4}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq}) \quad \mathrm{Kb}=\frac{\left[\mathrm{NH}_{4}^{+}\right] \cdot\left[\mathrm{OH}^{-}\right]}{\left[\mathrm{NH}_{3}\right]}
\end{aligned}
$$

The concentration of water ( $\sim 55.5$ ) is practically constant, especially if the solution is diluted.

## Reactions in gaseous phase.

The concentrations appearing in the expression of the equilibrium constant are usually given in mol/L(M), and the equilibrium constant is indicated by $\mathrm{K}_{C}$. However, the equilibrium constants for reactions that occur in the gas phase can be expressed in terms of partial pressures of reactants and products and the constant is then indicated as $\mathrm{K}_{\mathrm{p}}$ :

$$
\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g})=2 \mathrm{HI}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{C}}=\frac{[\mathrm{HI}]^{2}}{\left[\mathrm{H}_{2}\right] \cdot\left[\mathrm{I}_{2}\right]}
$$

From the gas state law (PV =nRT) we determine $[c]=n / V=P / R T$

$$
\mathrm{K}_{\mathrm{C}}=\frac{[\mathrm{HI}]^{2}}{\left[\mathrm{H}_{2}\right] \cdot\left[\mathrm{I}_{2}\right]}=\frac{\left(\frac{\mathrm{P}_{\mathrm{HI}}}{\mathrm{RT}}\right)^{2}}{\frac{\mathrm{P}_{\mathrm{H}_{2}}}{\mathrm{RT}} \cdot \frac{\mathrm{P}_{\mathrm{I}_{2}}}{\mathrm{RT}}}=\frac{\mathrm{P}_{\mathrm{HI}}^{2}}{\mathrm{P}_{\mathrm{H}_{2}} \cdot \mathrm{P}_{\mathrm{I}_{2}}}=\mathrm{K}_{\mathrm{P}}
$$

$K_{p}=K_{c}$ only if the constant is dimensionless. Eg. In the reaction of synthesis of ammonia
$\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g})=2 \mathrm{NH}_{3}(\mathrm{~g})$

$$
\mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{NH}_{3}\right]^{2}}{\left[\mathrm{~N}_{2}\right] \cdot\left[\mathrm{H}_{2}\right]^{3}}=\frac{\left(\frac{\mathrm{P}_{\mathrm{NH}_{3}}}{\mathrm{RT}}\right)^{2}}{\frac{\mathrm{P}_{\mathrm{N}_{2}}}{\mathrm{RT}} \cdot\left(\frac{\mathrm{P}_{\mathrm{H}_{2}}}{\mathrm{RT}}\right)^{3}}=\frac{\mathrm{P}_{\mathrm{NH}_{3}}^{2}}{\mathrm{P}_{\mathrm{N}_{2}} \cdot \mathrm{P}_{\mathrm{H}_{2}}^{3}} \cdot \frac{1}{(\mathrm{RT})^{2}} \rightarrow \mathrm{~K}_{\mathrm{P}}=\mathrm{K}_{\mathrm{C}}(\mathrm{RT})^{2}
$$

In the general case: $a A+b B=c C+d D$

$$
\mathrm{K}_{\mathrm{C}}=\frac{[\mathrm{C}]^{\mathrm{c}} \cdot[\mathrm{D}]^{\mathrm{d}}}{[\mathrm{~A}]^{\mathrm{a}} \cdot[\mathrm{~B}]^{\mathrm{b}}}=\frac{\mathrm{P}_{\mathrm{C}}^{\mathrm{c}} \cdot \mathrm{P}_{\mathrm{D}}^{\mathrm{d}}}{\mathrm{P}_{\mathrm{A}}^{\mathrm{a}} \cdot \mathrm{P}_{\mathrm{B}}^{\mathrm{b}}} \cdot(\mathrm{RT})^{\mathrm{a}+\mathrm{b-c-d}}=\mathrm{K}_{\mathrm{P}} \cdot(\mathrm{RT})^{\mathrm{a}+\mathrm{b-c-d}}
$$

$$
\mathrm{K}_{\mathrm{P}}=\mathrm{K}_{\mathrm{C}} \cdot(\mathrm{RT})^{\Delta \mathrm{n}}
$$

$$
\text { where } \Delta n=c+d-a-b
$$

Example 1: Write the expression for the equilibrium constant for each of the following reactions indicating their units:

$$
\begin{aligned}
& \text { a) } \mathrm{PCl}_{5}(\mathrm{~g})=\mathrm{PCl}_{3}(\mathrm{~g})+\mathrm{Cl}_{2}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{PCl}_{3}\right] \cdot\left[\mathrm{Cl}_{2}\right]}{\left[\mathrm{PCl}_{5}\right]}\{\mathrm{M}\} \\
& \text { b) } 2 \mathrm{H}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{H}_{2} \mathrm{O}\right]^{2}}{\left[\mathrm{H}_{2}\right]^{2} \cdot\left[\mathrm{O}_{2}\right]} \quad\left\{\mathrm{M}^{-1}\right\} \\
& \text { c) } \mathrm{CO}_{2}(\mathrm{~g})+\mathrm{C}(\mathrm{~s})=2 \mathrm{CO}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{C}}=\frac{[\mathrm{CO}]^{2}}{\left[\mathrm{CO}_{2}\right]}\{\mathrm{M}\} \\
& \text { d) } \mathrm{CH}_{3} \mathrm{COOH}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O} \text { (I) }=\mathrm{CH}_{3} \mathrm{COO}^{-}(\mathrm{aq})+\mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq}) \\
& K_{A}=\frac{\left[\mathrm{CH}_{3} \mathrm{COO}^{-}\right] \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]}{\left[\mathrm{CH}_{3} \mathrm{COOH}\right]} \quad\{M\} \quad K=\frac{\left[\mathrm{CH}_{3} \mathrm{COO}^{-}\right] \cdot\left[\mathrm{H}_{3} \mathrm{O}^{+}\right]}{[\mathrm{H} 2 \mathrm{O}]\left[\mathrm{CH}_{3} \mathrm{COOH}\right]}\{ \}
\end{aligned}
$$

Example 2: Write the expression for the equilibrium constant $K_{C}$ for each of the following reactions. What is the relationship is between $\mathrm{K}_{\mathrm{p}}$ and $\mathrm{K}_{C}$ ?
a) $3 \mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{O}_{3}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{O}_{3}\right]^{2}}{\left[\mathrm{O}_{2}\right]^{3}} \quad$ ozone $\mathrm{O}_{3}$ since $\quad K_{P}=K_{C} \cdot(R T)^{\frac{\Delta n}{e}} \Delta n=2-3=-1 \quad K_{P}=\frac{K_{C}}{R T}$
b) $2 \mathrm{SO}_{3}(\mathrm{~g})=2 \mathrm{SO}_{2}+\mathrm{O}_{2}(\mathrm{~g}) \quad \mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{SO}_{2}\right]^{2} \cdot\left[\mathrm{O}_{2}\right]}{\left[\mathrm{SO}_{3}\right]^{2}}$
since $\quad K_{P}=K_{C} \cdot(R T)^{\Delta n} \Delta n=3-2=1 \quad K_{P}=K_{C} R T$

Example 3: Myoglobin is a protein that binds $\mathrm{O}_{2}$ according to the reaction:

$$
\mathrm{Mb}(\mathrm{aq})+\mathrm{O}_{2}(\mathrm{aq})=\mathrm{MbO}_{2}(\mathrm{aq})
$$

At equilibrium: $[\mathrm{Mb}]=3.2 \mathrm{mM},\left[\mathrm{O}_{2}\right]=1.4 \mathrm{mM}$ $e\left[\mathrm{MbO}_{2}\right]=0.896 \mathrm{mM}$. Calculate $\mathrm{K}_{\mathrm{c}}$.


$$
\mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{MbO}_{2}\right]}{[\mathrm{Mb}] \cdot\left[\mathrm{O}_{2}\right]}=\frac{0.896}{3.2 \cdot 1.4}=0.2 \mu \mathrm{M}^{-1} \equiv 2 \cdot 10^{5} \mathrm{M}^{-1}
$$

Esemple 4: The reaction $\mathrm{PCl}_{5}(\mathrm{~g})=\mathrm{PCl}_{3}(\mathrm{~g})+\mathrm{Cl}_{2}(\mathrm{~g})$ has been studied at $250^{\circ} \mathrm{C}$. At equilibrium: $\left[\mathrm{PCl}_{5}\right]=4.2 \times 10^{-5} \mathrm{M},\left[\mathrm{PCl}_{3}\right]=1.3 \times 10^{-2} \mathrm{Me}\left[\mathrm{Cl}_{2}\right]$ $=3.9 \times 10^{-3} \mathrm{M}$. Calculate $\mathrm{K}_{c}$ and $\mathrm{K}_{p}$ for the reaction.

$$
\mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{PCl}_{3}\right] \cdot\left[\mathrm{Cl}_{2}\right]}{\left[\mathrm{PCl}_{5}\right]}=\frac{1.3 \cdot 10^{-2} \cdot 3.9 \cdot 10^{-3}}{4.2 \cdot 10^{-5}}=1.21 \mathrm{M}
$$

Since $\quad K_{P}=K_{C} \cdot(R T)^{\Delta n}$ and $\Delta n=2-1=1$
$\mathrm{K}_{\mathrm{P}}=\mathrm{K}_{\mathrm{C}} \mathrm{RT}=1.21 \cdot 0.082 \cdot(273.15+250)=51.8 \mathrm{~atm}$

## The equilibrium constant of a chemical reaction is a very useful parameter:

* if the ratio of the concentrations of reactants and products corresponds to the value of the equilibrium constant, the system is at equilibrium. If it has a different value, the system is not at equilibrium and it is possible to predict in which direction the reaction will proceed.
* The value of an equilibrium constant indicates whether a reaction is shifted towards the reactants or products.
* If the equilibrium concentrations of species involved in the reaction are known, the equilibrium concentration of other species can be calculated from the expression of K .
* If the initial concentrations of some species are known, the equilibrium concentration of products and reactants can be calculated from the expression of $K$.
$K \gg 1$ : the reaction is shifted toward the products. Equilibrium concentrations of the products are larger than concentrations of the reactants.
$\mathrm{NO}(\mathrm{g})+\mathrm{O}_{3}(\mathrm{~g})=\mathrm{NO}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g})$

$$
\mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{NO}_{2}\right] \cdot\left[\mathrm{O}_{2}\right]}{[\mathrm{NO}] \cdot\left[\mathrm{O}_{3}\right]}=6 \cdot 10^{34} \text { a } 25^{\circ} \mathrm{C}
$$



The large value of K indicates that $\left[\mathrm{NO}_{2}\right]\left[\mathrm{O}_{3}\right] \gg[\mathrm{NO}]\left[\mathrm{O}_{2}\right]$
$K \ll 1$ : the reaction is shifted toward the reagents. Equilibrium concentrations of reactants are larger than concentrations of the products.

$$
\begin{aligned}
& 3 \mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{O}_{3}(\mathrm{~g}) \\
& \mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{O}_{3}\right]^{2}}{\left[\mathrm{O}_{2}\right]^{3}}=6.2 \cdot 10^{-58} \text { a } 25^{\circ} \mathrm{C}
\end{aligned}
$$



The small value of K indicates that $\left[\mathrm{O}_{3}\right]^{2} \ll\left[\mathrm{O}_{2}\right]^{3}$

## Equilibrium constants for some reactions

| reaction | $\mathrm{K}_{\mathrm{C}}$ a $25^{\circ} \mathrm{C}$ | Reaction is shifted toward |
| :---: | :---: | :---: |
| Reaction of combination with non metals |  |  |
| $\mathrm{S}(\mathrm{s})+\mathrm{O}_{2}(\mathrm{~g})=\mathrm{SO}_{2}(\mathrm{~g})$ | $4.2 \cdot 10^{52} \mathrm{M}^{-1}$ | $\mathrm{K} \gg 1$, products |
| $2 \mathrm{H}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ | $3.1 \cdot 10^{81} \mathrm{M}^{-1}$ | $\mathrm{K} \gg 1$, products |
| $\mathrm{N}_{2}(\mathrm{~g})+3 \mathrm{H}_{2}(\mathrm{~g})=2 \mathrm{NH}_{3}(\mathrm{~g})$ | $3.5 \cdot 10^{8} \mathrm{M}^{-2}$ | $\mathrm{K}>1$, products |
| $\mathrm{N}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{NO}(\mathrm{g})$ | $4.2 \cdot 10^{-3}$ * | $\mathrm{K}<1$, reagents |
| Ionization reactions of acids and bases |  |  |
| $\mathrm{HCO}_{2} \mathrm{H}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l})=\mathrm{HCO}_{2}^{-}(\mathrm{aq})+\mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})$ | $1.8 \cdot 10^{-4} \mathrm{M}$ | $\mathrm{K}<1$, reagents |
| $\mathrm{CH}_{3} \mathrm{CO}_{2} \mathrm{H}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l})=\mathrm{CH}_{3} \mathrm{CO}_{2}^{-}(\mathrm{aq})+\mathrm{H}_{3} \mathrm{O}^{+}$(aq) | $1.8 \cdot 10^{-5} \mathrm{M}$ | $\mathrm{K}<1$, reagents |
| $\mathrm{H}_{2} \mathrm{CO}_{3}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l})=\mathrm{HCO}_{3}^{-}(\mathrm{aq})+\mathrm{H}_{3} \mathrm{O}^{+}(\mathrm{aq})$ | $4.2 \cdot 10^{-7} \mathrm{M}$ | $\mathrm{K}<1$, reagents |
| $\mathrm{NH}_{3}(\mathrm{aq})+\mathrm{H}_{2} \mathrm{O}(\mathrm{l})=\mathrm{NH}_{4}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})$ | $1.8 \cdot 10^{-5} \mathrm{M}$ | $\mathrm{K}<1$, reagents |

## The reation quotient $Q$

The equilibrium constant $K$ has a numerical value defined when reactants and products are at equilibrium. When reactants and products are not at equilibrium it is useful to calculate the reaction quotient $Q$ :

## $a A+b B=c C+d D$

$$
\mathrm{Q}=\frac{[\mathrm{C}]^{\mathrm{c}} \cdot[\mathrm{D}]^{\mathrm{d}}}{[\mathrm{~A}]^{\mathrm{a}} \cdot[\mathrm{~B}]^{\mathrm{b}}}
$$

The concentrations of reagents in the expression of $Q$ are those present in the system at any time since the beginning of the reaction until equilibrium is reached, when $Q=K$. The knowledge of $Q$ is useful for 2 reasons:

1) allows to determine if the system is a $\dagger$ equilibrium $(Q=K)$ or not $(Q \neq K)$
2) confronting $Q$ with $K$ we know in which direction the reaction will proceed.

If $Q<K$, equilibrium will be reached by transforming reagents into products
If $Q>K$, equilibrium will be reached by transforming products into reagents

In the isomerization of butane $\left(\mathrm{C}_{4} \mathrm{H}_{10}\right)$ to isobutane $\left(\mathrm{C}_{4} \mathrm{H}_{10}\right)$ a 298 K :
butano $(\mathrm{g}) \rightleftarrows$ isobutano ( g )


Example 5: At 2000 K , the equilibrium constant for the reaction

$$
\mathrm{N}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{NO}(\mathrm{~g})
$$

$\mathrm{K}=4.0 \times 10^{-4}$. If $\left[\mathrm{N}_{2}\right]=0.5 \mathrm{M},\left[\mathrm{O}_{2}\right]=0.25 \mathrm{Me}[\mathrm{NO}]=0.042 \mathrm{M}$, determine if the reaction is at equilibrium, if it is not the case determine in which direction it will proceed.

$$
\mathrm{Q}=\frac{[\mathrm{NO}]^{2}}{\left[\mathrm{~N}_{2}\right]\left[\mathrm{O}_{2}\right]}=\frac{0.042^{2}}{0.5 \cdot 0.25}=1.41 \cdot 10^{-3}>\mathrm{K}
$$

The reaction is not at equilibrium $\mathrm{Q}>\mathrm{K}$.
Equilibrium can be reached transforming NO into $\mathrm{N}_{2}$ and $\mathrm{O}_{2}$.

$$
\mathrm{N}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{NO}(\mathrm{~g})
$$

Example 6: At 298 K, the equilibrium constant for the reaction

$$
2 \mathrm{NO}_{2}(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})
$$

is $\mathrm{K}=170$. If $\left[\mathrm{NO}_{2}\right]=0.015 \mathrm{Me}\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]=0.025 \mathrm{M}$, is the reaction at equilibrium? If it is not so, in which direction will it proceed?

$$
\mathrm{Q}=\frac{\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]}{\left[\mathrm{NO}_{2}\right]^{2}}=\frac{0.025}{0.015^{2}}=111.1<\mathrm{K}
$$

The reaction is not at equilibrium $\operatorname{sinc} \mathrm{Q}<\mathrm{K}$.
Equibrium can be achieved by production of $\mathrm{N}_{2} \mathrm{O}_{4}$.

$$
2 \mathrm{NO}_{2} \overrightarrow{(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})}
$$

How to calculate the equilibrium constant
If the concentrations at equilibrium of some of the compounds involved in the reaction are known, the equilibrium concentration of other species can be calculated from the balanced chemical equation.
E.g. $\mathrm{T}=1000 \mathrm{~K} \quad 2 \mathrm{SO}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g}) \longleftrightarrow 2 \mathrm{SO}_{3}(\mathrm{~g})$

Before equilibrium we have $1 \mathrm{~mol} / \mathrm{L}$ of $\mathrm{SO}_{2}$ and $1 \mathrm{~mol} / \mathrm{L}$ of $\mathrm{O}_{2}$. At equilibrium we have $0.925 \mathrm{~mol} / \mathrm{L}^{\text {of }} \mathrm{SO}_{3}$.
Calculate the equilibrium concentration of all compounds and $\mathrm{K}_{\mathrm{C}}$.

| equation | $2 \mathrm{SO}_{2}(\mathrm{~g})$ | $+\mathrm{O}_{2}(\mathrm{~g})=2 \mathrm{SO}_{3}(\mathrm{~g})$ |  |
| :--- | :---: | :---: | :---: |
| Initial <br> concentrations (M) | 1.0 | 1.0 | 0 |
| variation (M) | -0.925 | $-0.925 / 2$ | +0.925 |
| Concentrations at <br> equilibrium (M) | 0.075 | 0.5375 | 0.925 |

$$
\mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{SO}_{3}\right]^{2}}{\left[\mathrm{SO}_{2}\right]^{2} \cdot\left[\mathrm{O}_{2}\right]}=\frac{0.925^{2}}{0.075^{2} \cdot 0.5375}=283 \mathrm{M}^{-1}
$$

$$
\begin{gathered}
a x^{2}+b x+c=0 \\
x_{1,2}=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
\end{gathered}
$$

## Equilibrium constant:

In many cases K and the initial concentrations of reagents are known and we need to calculate the concentrations of species at equilibrium.
E.g. In the reaction:

$$
\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g})=2 \mathrm{HI}(\mathrm{~g})
$$

At $425^{\circ} \mathrm{C}, \mathrm{K}=55.64$.
If we mix in a 0.5 L container 1.0 mole of $\mathrm{H}_{2}$ and 1.0 mole of $\mathrm{I}_{2}$ at $425^{\circ} \mathrm{C}$, what are the equilibrium concentrations of $\mathrm{H}_{2}, \mathrm{I}_{2} \mathrm{e} \mathrm{HI}$ ?

$$
\mathrm{K}_{\mathrm{C}}=\frac{[\mathrm{HI}]^{2}}{\cline { 2 - 4 }} \begin{array}{|c|c|c|c|}
\left.\hline \mathrm{H}_{2}\right] \cdot\left[\mathrm{I}_{2}\right] & \mathrm{H}_{2}(\mathrm{~g}) & + & \mathrm{I}_{2}(\mathrm{~g}) \\
\cline { 2 - 4 } & 1.0 / 0.5=2 & 1.0 / 0.5=2 & 0 \\
\cline { 2 - 4 } & ? & \mathrm{HI}(\mathrm{~g}) \\
\cline { 2 - 4 } & ? & ? & ? \\
\cline { 2 - 4 } & ? & ? & ? \\
\cline { 2 - 4 } & & ? & \\
\hline
\end{array}
$$

If x moles of $\mathrm{H}_{2}$ and $\mathrm{I}_{2}$ react, we will have 2 x moles of HI :

| equation | $\mathrm{H}_{2}(\mathrm{~g})+\mathrm{I}_{2}(\mathrm{~g})=2 \mathrm{HI}(\mathrm{g})$ |  |  |
| :---: | :---: | :---: | :---: |
| Initial concentrations (M) | $1.0 / 0.5=2$ | $1.0 / 0.5=2$ | 0 |
| variation (M) | -x | -x | $+2 \mathrm{x}$ |
| Concentrations at equilibrium (M) | $2-x$ | $2^{-x}$ | 2 x |

$$
\begin{aligned}
& \mathrm{K}=\frac{[\mathrm{HI}]^{2}}{\left[\mathrm{H}_{2}\right] \cdot\left[\mathrm{I}_{2}\right]}=\frac{(2 \mathrm{x})^{2}}{(2-\mathrm{x}) \cdot(2-\mathrm{x})} \rightarrow(\mathrm{K}-4) \mathrm{x}^{2}-4 \mathrm{Kx}+4 \mathrm{~K}=0 \\
& \mathrm{x}=\frac{2 \mathrm{~K}-4 \sqrt{\mathrm{~K}}}{\mathrm{~K}-4}=\frac{2 \cdot 55.64-4 \sqrt{55.64}}{55.64-4}=1.577 \mathrm{M}
\end{aligned}
$$

Concentrations at equilibrium are: $\left[\mathrm{H}_{2}\right]=\left[\mathrm{I}_{2}\right]=2-1.577=0.423 \mathrm{Me}[\mathrm{HI}]=$ 3.154 M.

## Perturbations of a chemical equilibrium: Le Chatelier's principle

The equilibrium between reagents and products can be perturbed in three ways:

- Changing temperature

H. L. Le Chatelier 1850-1936)
- Changing the concentration of a reactant or product
- Changing volume (for reactions in the gaseous phase)

Le Chatelier' principle: If a chemical system at equilibrium experiences a change in concentration, temperature, volume, or partial pressure, then the equilibrium shifts to counteract the imposed change and a new equilibrium is established.


## Effect of the addition or removal of a reactant or product

If the concentration of a reactant or a product is changed (at $T=$ constant), the reaction moves toward a new equilibrium in which the reaction quotient is still equal to K .

In the isomerization reaction of butane $\left(\mathrm{C}_{4} \mathrm{H}_{10}\right)$ to isobutane $\left(\mathrm{C}_{4} \mathrm{H}_{10}\right)$ at 298 K :

to


$$
\mathrm{Q}=5 / 2=\mathrm{K}
$$

$\mathrm{Q}=12 / 2>\mathrm{K}$
$\mathrm{Q}=10 / 4=\mathrm{K}^{3}$

## Effect of volume changes on equilibrium in gaseous phase

In an equilibrium that involves gases, what happens to the concentrations or partial pressures if the size of the container is changed?


What happens to this equilibrium if the volume of the container is suddenly halved at constant temperature?

According to Boyle's law, pressure doubles ( $P_{1} \times V_{1}=P_{2} \times V_{2}$, con $V_{2}=V_{1} / 2$ ) and also concentrations ( $c=n / V=P /(R T)$ )


We have an equlibrium with 0.2 mol of $\mathrm{NO}_{2}$ e 9.86 mol of $\mathrm{N}_{2} \mathrm{O}_{4}$ in 1 L . After compression pression is halved and concentrations are doubled :

$$
\mathrm{Q}=\frac{2 \cdot\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]}{\left(2 \cdot\left[\mathrm{NO}_{2}\right]\right)^{2}}=\frac{\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]}{2 \cdot\left[\mathrm{NO}_{2}\right]^{2}}=123.25 \mathrm{M}^{-1}<\mathrm{K}_{\mathrm{C}} \quad 2 \mathrm{NO}_{2}(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})
$$

Since $Q<K$, the amount of product must increase and the amount of reagent must decrease to be again at equilibrium.
The total number of molecules is decreased.

nuovo $\xrightarrow{\text { equilibrio }}$


| equation | $2 \mathrm{NO}_{2}(\mathrm{~g})=$ | $\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})$ |
| :--- | :---: | :---: |
| Initial concentrations <br> $(M)$ | $0.2 / 1$ | $9.86 / 1$ |
| Concentrations after <br> compression (M) | $0.2 / 0.5=$ <br> 0.4 | $9.86 / 0.5=$ <br> 19.72 |
| variation (M) | -x | $+\mathrm{x} / 2$ |
| Concentrations at <br> equilibrium (M) | $0.4-\mathrm{x}$ | $19.72+\mathrm{x} / 2$ |

$$
2 \mathrm{NO}_{2}(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})
$$

$$
\mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]}{\left[\mathrm{NO}_{2}\right]^{2}}=\frac{19.72+\frac{\mathrm{x}}{2}}{(0.4-\mathrm{x})^{2}}=246.5 \mathrm{M}^{-1} \rightarrow \mathrm{x}=0.1167 \mathrm{M}
$$

The new equilibrium concentrations are:

$$
\left[\mathrm{NO}_{2}\right]=0.4-0.1167=0.2833 \mathrm{M} \mathrm{e}\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]=19.72+0.1167 / 2=19.78 \mathrm{M}
$$

Number of mole before compression $=(0.2+9.86) \times 1=10.06$
Number of molecules after compression, at equilibrium $=(0.2833+19.78) \times 0.5=10.03$

## IN CONCLUSION

- for a reaction involving gases, the perturbation produced by a volume decrease (or increase in pressure) is balanced by a shift in the direction that results in fewer molecules of gas.
- an increase in volume (or pressure drop) produces the opposite effect: the equilibrium shifts in the direction that results in a larger number of molecules.
- For a reaction in which the number of molecules does not change (ex. $A+B=2 C$ ), a change in volume (or pressure) can not perturb the equilibrium.

$$
\mathrm{N}_{2}(\mathrm{~g})+\begin{gathered}
\stackrel{\Delta \mathrm{V}<0(\Delta \mathrm{P}>0)}{\stackrel{+}{\mathrm{H}}(\mathrm{~g})=2} \mathrm{NH}_{3}(\mathrm{~g}) \\
\Delta \mathrm{V}>0(\Delta \mathrm{P}<0)
\end{gathered}
$$

## Exothermic and endothermic reactions the solubilization of strong

 electrolytes in $\mathrm{H}_{2} \mathrm{O}$

An exothermic reaction
$\mathrm{NaOH}(s) \longrightarrow \mathrm{Na}^{+}(\mathrm{aq})+\mathrm{OH}^{-}(\mathrm{aq})+$ heat
$\Delta H_{\text {sol }}=H_{\text {products }}-H_{\text {reagents }}<0=-44.4 \mathrm{~kJ} / \mathrm{mol}$


An endothermic reaction
$\mathrm{NH}_{4} \mathrm{NO}_{3}(\mathrm{~s})+$ heat $\quad \mathrm{NH}_{4}^{+}(\mathrm{aq})+\mathrm{NO}_{3}^{-}(\mathrm{aq})$
$\Delta H_{\text {sol }}=H_{\text {products }}-H_{\text {reagents }}>0=+25.7 \mathrm{~kJ} / \mathrm{mol}$

## Effect of temperature on equlibrium

It is possible to make a qualitative prediction of the effect of a change in temperature depending on whether the reaction is exothermic or endothermic. The value of the equilibrium constant changes.
$\mathrm{N}_{2}(\mathrm{~g})+\mathrm{O}_{2}(\mathrm{~g})+$ heat $=2 \mathrm{NO}(\mathrm{g}) \quad \Delta \mathrm{H}_{\text {reaction }}=+180.6 \mathrm{~kJ} / \mathrm{mol} \quad \mathrm{K}=\frac{[\mathrm{NO}]^{2}}{\left[\mathrm{~N}_{2}\right] \cdot\left[\mathrm{O}_{2}\right]}$
The reaction is endothermic: it is necessary to provide heat.

| $\mathrm{T}(\mathrm{K})$ | K | $\ln (\mathrm{K})$ |
| :---: | :---: | :---: |
| 200 | $6.77 \times 10^{-48}$ | -108.61 |
| 298 | $2.20 \times 10^{-32}$ | -72.89 |
| 900 | $3.30 \times 10^{-11}$ | -24.13 |
| 2300 | $7.91 \times 10^{-5}$ | -9.44 |
|  |  | van' + Hoff <br> equation |

$$
\ln K=-\frac{\Delta H_{\text {reaction }}}{R T}
$$

$$
2 \mathrm{NO}_{2}(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})+\text { heat } \quad \Delta \mathrm{H}_{\text {reazione }}=-17.08 \mathrm{~kJ} / \mathrm{mol}
$$

$$
\mathrm{K}=\frac{\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]}{\left[\mathrm{NO}_{2}\right]^{2}}
$$

The reaction is exothermic: it involves the release of heat.


- when the temperature of a system at equilibrium increases, it will shift towards the direction that absorbes heat, i.e. the endothermic direction.
- when the temperature of a system at equilibrium decreases, it will shift towards the direction that releases heat, i.e. the exothermic direction.
- changing the temperature will change concentrations at equilibrium and it will change K

We can consider the heat " $q$ " as a "reagent" or a " product".


## VARIATIONS IN CONCENTRATION

Increasing the concentration of a compound in a reaction at equilibrium determines a shift in the direction that consumes part of the added material (Le Chatelier).

$$
\mathrm{N}_{2}+3 \mathrm{H}_{2} \leftrightarrows 2 \mathrm{NH}_{3}
$$

## VARIATIONS IN PRESSURE

If an increase in pressure is applied, the balance shifts to achieve its decrease: for this reaction ammonia is formed since products lead to an increase in particles. The equilibrium will shift to the right.

## Effects of perturbations to the equilibrium

| perturbation | Change to achieve equilibrium | Effect on the reaction | Effect on K |
| :---: | :---: | :---: | :---: |
| Reactions in solid, liquid and gaseous phase |  |  |  |
| Increase in $T$ $(\Delta \mathrm{T}>0)$ | Heat is absorbed by the system | Shift towards endothermic direction | change |
| $\begin{aligned} & \text { Decrease in } \mathrm{T} \\ & (\Delta \mathrm{~T}<0) \end{aligned}$ | Heat is released by the system | Shift towards exothermic direction | change |
| Reagents added | The reagent is partially consumed | [products] increase | No change |
| Products added | The product is partially consumed | [reagents] increase | No change |
| Reaction in gaseous phase |  |  |  |
| V decreases <br> $P$ increases | Pressure increases | The reaction shifts to decrease the number of molecules | No change |
| P decreases <br> V increases | Pressure decreases | The reaction shifts to increase the number of molecules | No change |

## EQUILIBRIUM AND DISSOCIATION DEGREE

$\mathrm{PCl}_{5(\mathrm{~g})} \rightleftarrows \mathrm{PCl}_{3(\mathrm{~g})}+\mathrm{Cl}_{2(\mathrm{~g})} \quad \mathrm{K}_{\mathrm{p}}=\frac{\mathrm{p}_{\mathrm{PCl}_{3}} \mathrm{p}_{\mathrm{Cl}_{2}}}{\mathrm{p}_{\mathrm{PCl}_{5}}}=\frac{\mathrm{x}^{2}}{1-\mathrm{x}}$

If 1 mole of $\mathrm{PCl}_{5}$ is present and, at a given T , it has a dissociation degree $x=30 \%(=0,3)$, at equilibrium we will have:
$0,3(x) n$ di $\mathrm{PCl}_{3} ; \quad 0,3(x) n$ di $\mathrm{Cl}_{2} ; \quad 0,7(1-x) \mathrm{ndi} \mathrm{PCl}_{5}$
$K=(0,3)^{2} / 0,7$

## Ostwald's Law (law of dilution)

In a dissociation equilibrium where $c$ is the initial concentration:

$$
\begin{aligned}
& A B \underset{C(1-\alpha)}{\rightleftarrows} \quad c \alpha+B \\
& c \alpha
\end{aligned}
$$

Therefore $K_{c}=\frac{c^{2} \alpha^{2}}{c(1-\alpha)}=\frac{c \alpha^{2}}{(1-\alpha)}$

If $\alpha \ll 1$ then $1-\alpha$ can be approximated as $=1$

$$
K_{c}=c \alpha^{2}
$$

Depending of the dissociation equilibrium, Ostwald's law can change formulation:

$$
\underset{c(1-\alpha)}{A_{2} \rightleftarrows} \underset{2 c \alpha}{\rightleftarrows} \underset{2}{2}
$$

In this case $K_{c}=\frac{4 c^{2} \alpha^{2}}{c(1-\alpha)}=\frac{4 c \alpha^{2}}{(1-\alpha)}$

## Why Ostwald's law or law of dilution?

The law explains why increasing dilution (=concentration decreases) the dissociation degree increases.

$$
K_{c}=\frac{c \alpha^{2}}{(1-\alpha)}
$$

By increasing $\alpha K_{c}$ stays constant as $c$ decreases.

Example : The K for molecular iodine dissociation is $1000{ }^{\circ} \mathrm{C}$

$$
\mathrm{I}_{2}(\mathrm{~g})=2 \mathrm{I}(\mathrm{~g})
$$

is $\mathrm{K}=0.00376 \mathrm{M}$. What are the equilibrium concentrations if initially we have 0.105 moles of $\mathrm{I}_{2}$ in 12.3 L at $1000^{\circ} \mathrm{C}$ ?

| equation | $\mathrm{I}_{2}(\mathrm{~g})=2 \mathrm{I}(\mathrm{g})$ |  |
| :--- | :---: | :---: |
| Initial concentration <br> $(\mathrm{M})$ | $0.105 / 12.3$ <br> $=0.0085$ | 0 |
| variation (M) | -x | +2 x |
| Equilibrium <br> concentration(M) | $0.0085-\mathrm{x}$ | 2 x |

$$
\begin{gathered}
\text { If } \mathrm{c} \text { is the initial concentration of } \\
\mathrm{I}_{2}, \mathrm{c}=0.105 / 12.3=0.0085 \mathrm{M} \\
\mathrm{~K}=\frac{[\mathrm{I}]^{2}}{\left[\mathrm{I}_{2}\right]}=\frac{(2 \mathrm{x})^{2}}{(\mathrm{c}-\mathrm{x})} \\
4 \mathrm{x}^{2}+\mathrm{Kx}-\mathrm{Kc}=0
\end{gathered}
$$

$x=\frac{-K+\sqrt{K^{2}+16 K c}}{8}=\frac{-0.00376+\sqrt{0.00376^{2}+16 \cdot 0.00376 \cdot 0.0085}}{8}=0.0024 \mathrm{M}$
At equilibrium: $\left[\mathrm{I}_{2}\right]=0.0085-0.0024=0.0061 \mathrm{Me}$ e $[\mathrm{I}]=2 \times 0.0024=0.0048 \mathrm{M}$

Example : The K for molecular iodine dissociation is $1000{ }^{\circ} \mathrm{C}$

$$
\mathrm{I}_{2}(\mathrm{~g})=2 \mathrm{I}(\mathrm{~g})
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| equation | $\mathrm{I}_{2}(\mathrm{~g})=2 \mathrm{I}(\mathrm{g})$ |  |
| :--- | :---: | :---: |
| Initial concentration <br> $(\mathrm{M})$ | $0.105 / 12.3$ <br> $=0.0085$ | 0 |
| variation (M) | -x | +2 x |
| Equilibrium <br> concentration(M) | $0.0085-\mathrm{x}$ | 2 x |

$$
\begin{gathered}
\text { If } \mathrm{c} \text { is the initial concentration of } \\
\mathrm{I}_{2}, \mathrm{c}=0.105 / 12.3=0.0085 \mathrm{M} \\
\mathrm{~K}=\frac{[\mathrm{I}]^{2}}{\left[\mathrm{I}_{2}\right]}=\frac{(2 \mathrm{x})^{2}}{(\mathrm{c}-\mathrm{x})} \\
4 \mathrm{x}^{2}+\mathrm{Kx}-\mathrm{Kc}=0
\end{gathered}
$$

$x=\frac{-K+\sqrt{K^{2}+16 K c}}{8}=\frac{-0.00376+\sqrt{0.00376^{2}+16 \cdot 0.00376 \cdot 0.0085}}{8}=0.0024 \mathrm{M}$
At equilibrium: $\left[\mathrm{I}_{2}\right]=0.0085-0.0024=0.0061 \mathrm{Me}$ e $[\mathrm{I}]=2 \times 0.0024=0.0048 \mathrm{M}$

| equation | $2 \mathrm{NO}_{2}(\mathrm{~g})=$ | $\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})$ |
| :--- | :---: | :---: |
| Initial concentrations <br> (M) | $0.2 / 1$ | $9.86 / 1$ |
| Concentrations after <br> pressure increase(M) |  |  |
| variation (M) |  |  |
| New equilibrium <br> concentration (M) |  |  |

$2 \mathrm{NO}_{2}(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})$

The volume is halved (conc is doubled) by doubling the pressure

| equation | $2 \mathrm{NO}_{2}(\mathrm{~g})=$ | $\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})$ |
| :--- | :---: | :---: |
| Initial concentrations <br> $(M)$ | $0.2 / 1$ | $9.86 / 1$ |
| Concentrations after <br> pressure increase(M) | $0.2 / 0.5=$ <br> 0.4 | $9.86 / 0.5=$ <br> 19.72 |
| variation (M) | -x | $+\mathrm{x} / 2$ |
| New equilibrium <br> concentration $(\mathrm{M})$ | $0.4-\mathrm{x}$ | $19.72+\mathrm{x} / 2$ |

$2 \mathrm{NO}_{2}(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})$

$$
K_{C}=\frac{\left[N_{2} O_{4}\right]}{\left[N O_{2}\right]^{2}}=246.5 \mathrm{M}^{-1}
$$

$$
K_{C}=\frac{\left[N_{2} O_{4}\right]}{\left[N O_{2}\right]^{2}}=\frac{19.72+\frac{x}{2}}{(0.4-x)^{2}}=246.5 \mathrm{M}^{-1}
$$

$$
\frac{19.72+\frac{x}{2}}{(0.4-x)^{2}}=246.5
$$

$$
\begin{aligned}
& 246.5(0.4-x)^{2}=19.72+0.5 x \\
& 246.5\left(0.16-0.8 x+x^{2}\right)=19.72+0.5 x \\
& 39.44+-197.2 x+246.5 x^{2}-19.72+0.5 x=0 \\
& 246.5 x^{2}-197.7 x+19.72=0 \\
& x=\frac{197.7 \pm \sqrt{39085.29-19443.92}}{493} \\
& x=\frac{197.7 \pm \sqrt{19641.37}}{493}=\frac{197.7 \pm 140.1}{493} \\
& =0.685,0.117
\end{aligned}
$$

| equation | $2 \mathrm{NO}_{2}(\mathrm{~g})$ | $=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})$ |
| :--- | :---: | :---: |
| Initial concentrations <br> $(M)$ | $0.2 / 1$ | $9.86 / 1$ |
| Concentrations after <br> pressure increase(M) | $0.2 / 0.5=$ <br> 0.4 | $9.86 / 0.5=$ <br> 19.72 |
| variation (M) | -x | $+\mathrm{x} / 2$ |
| New equilibrium <br> concentration $(M)$ | $0.4-\mathrm{x}$ | $19.72+\mathrm{x} / 2$ |

$$
2 \mathrm{NO}_{2}(\mathrm{~g})=\mathrm{N}_{2} \mathrm{O}_{4}(\mathrm{~g})
$$

## $0.685,0.117$

$$
\mathrm{K}_{\mathrm{C}}=\frac{\left[\mathrm{N}_{2} \mathrm{O}_{4}\right]}{\left[\mathrm{NO}_{2}\right]^{2}}=\frac{19.72+\frac{\mathrm{x}}{2}}{(0.4-\mathrm{x})^{2}}=246.5 \mathrm{M}^{-1} \rightarrow \mathrm{x}=0.1167 \mathrm{M}
$$

The new equilibrium concentrations are:

$$
\left[\mathrm{NO}_{2}\right]=0.4-0.1167=0.2833 \mathrm{M} \mathrm{e}\left[\mathrm{~N}_{2} \mathrm{O}_{4}\right]=19.72+0.1167 / 2=19.78 \mathrm{M}
$$

Number of moles before perturbation $=(0.2+9.86) \times 1=10.06$
Number of moles afre perturbation $=(0.2833+19.78) \times 0.5=10.03$

