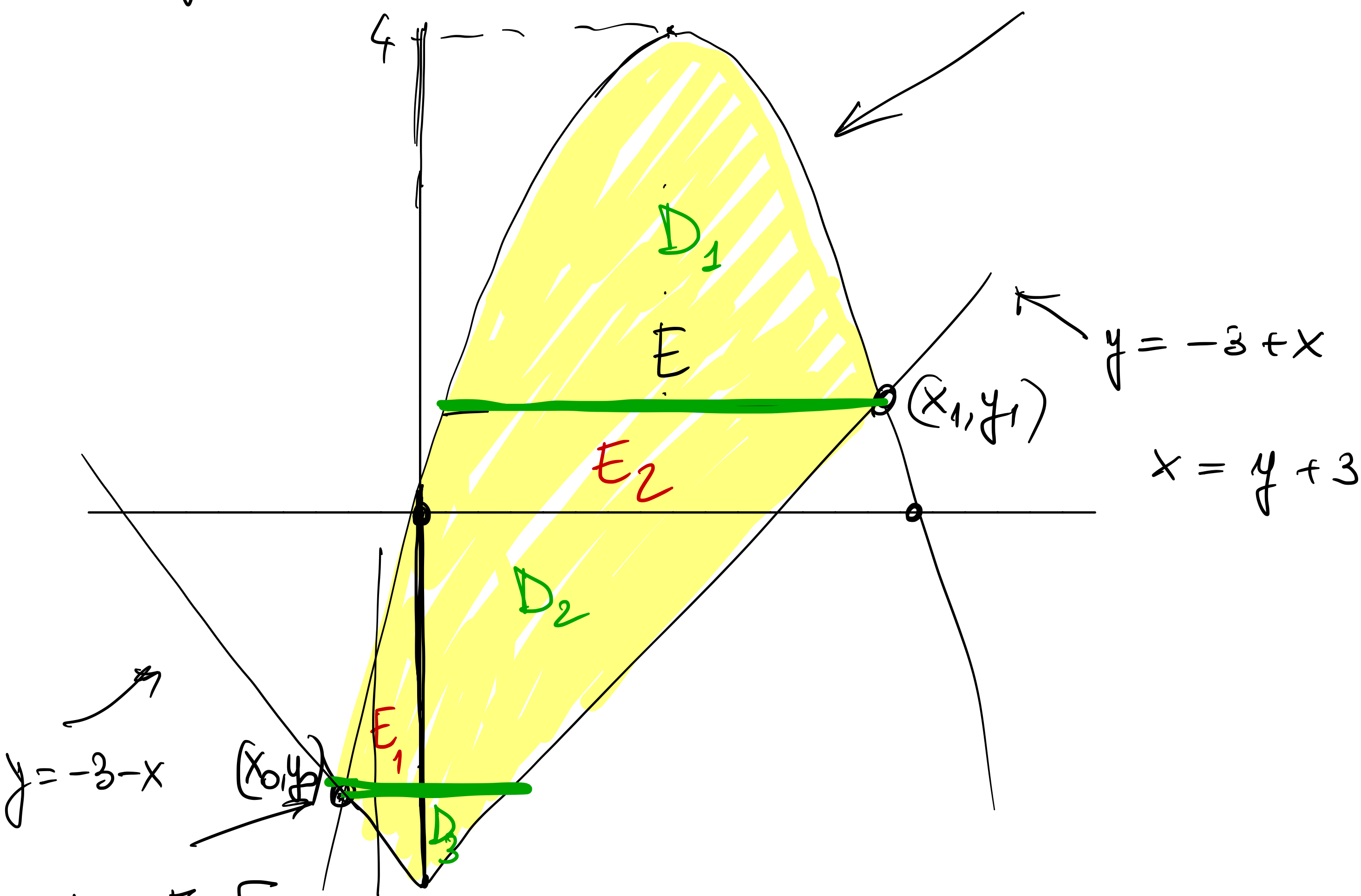
Sia $E = \{(x,y) : -3+|x| \leq y \leq -x^2+4x\}$

Calcolare $\iint_E xy \, dx\,dy$

e poi impostare l'integrale cambiando l'ordine di integrazione (senza ricalcolarlo).



$x = \frac{5-\sqrt{37}}{2} = x_0 \qquad y_0 = -3-x_0$

$$\begin{cases} y = -3-x \\ y = -x^2+4x \end{cases} \qquad -x^2+4x = -3-x$$

$$x^2-5x-3=0$$

$$x = \frac{5 \pm \sqrt{25+12}}{2} = \frac{5\pm\sqrt{37}}{2}$$

$$\begin{cases} y = -3+x \\ y = -x^2+4x \end{cases} \qquad -x^2+4x = -3+x$$

$$x^2-3x-3=0$$

$$x^2-3x-3=0$$

$$x = \frac{3 \pm \sqrt{9+12}}{2} = \frac{3 \pm \sqrt{21}}{2}$$

$$x_1 = \frac{3+\sqrt{21}}{2}$$

$$y_1 = 3 + x_1$$

$$\iint_E xy \, dx dy = \iint_{E_1} xy \, dx dy + \iint_{E_2} xy \, dx dy =$$

$$= \int_{x_0}^{0} dx \int_{-3-x}^{-x^2+4x} dy \; x y + \int_{0}^{x_1} dx \, x \int_{-3+x}^{-x^2+4x} dy \; y =$$

$$= \frac{1}{2} \int_{x_0}^{0} dx \; x \left[\left(-x^2+4x\right)^2 - \left(-3-x\right)^2 \right] +$$

$$+ \frac{1}{2} \int_{0}^{x_1} dx \; x \left[\left(-x^2+4x\right)^2 - \left(-3+x\right)^2 \right] = \text{facile}$$

Cambiamo ordine di integrazione

$$\iint_E xy\,dx\,dy = \iint_{D_1} xy\,dx\,dy + \iint_{D_2} \cdots + \iint_{D_3} \cdots$$

$$= \int_{y_1}^{4} dy \int_{2-\sqrt{4-y}}^{2+\sqrt{4-y}} dx \;\; x\,y \; + \; \int_{y_0}^{y_1} dy \int_{2-\sqrt{4-y}}^{3+y} dx \;\; x\,y \; +$$

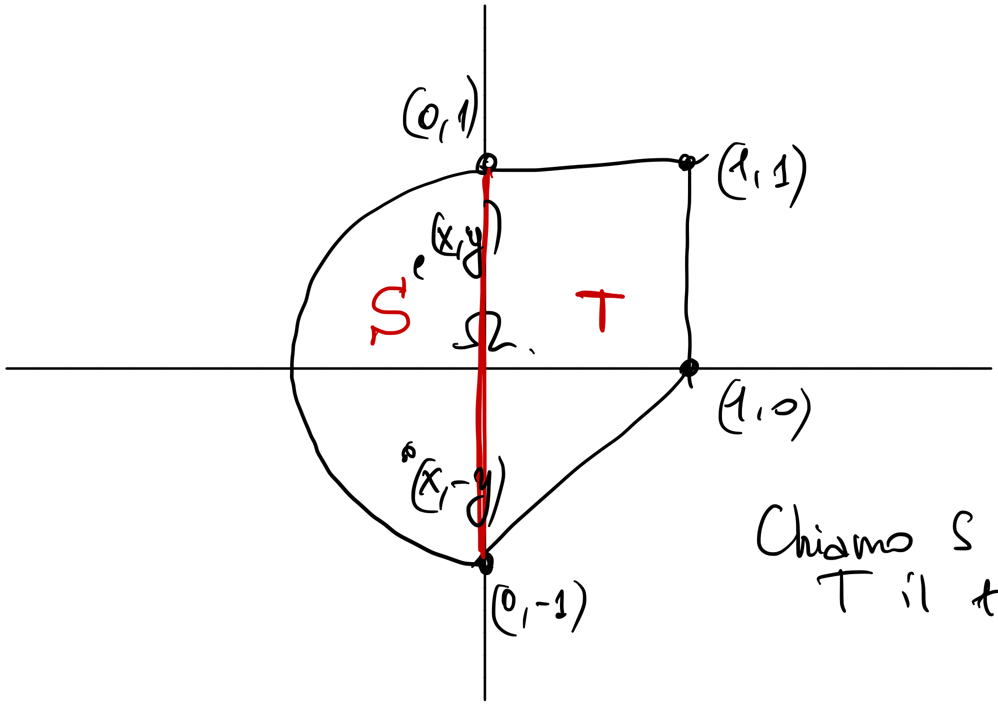
$$+ \int_{-3}^{y_0} dy \int_{-3-y}^{3+y} dx \;\; x\,y$$

$$y = -x^2+4x \qquad\qquad x^2-4x+y=0$$

$$x = 2 \pm \sqrt{4-y}$$

$$\iint_\Omega xy \, dx\,dy$$

Ω è l'unione del trapezio di vertici $(0,-1)$, $(1,0)$, $(1,1)$, $(0,1)$ e del semicerchio di centro l'origine, raggio 1 e ascisse negative.



Chiamo S il semicerchio
T il trapezio.

$$\iint_\Omega xy\, dx\,dy = \iint_S xy\, dx\,dy + \iint_T xy\, dx\,dy$$

coord. polari:

$$\iint_S xy\, dx\,dy = \int_{\pi/2}^{3/2\pi} d\theta \int_0^1 d\rho\, \rho^2 \cos\theta \,\mathrm{sen}\,\theta\, \rho = \left(\int_0^1 d\rho\, \rho^3\right)\underbrace{\left(\int_{\pi/2}^{3/2\pi} \cos\theta\,\mathrm{sen}\,\theta\, d\theta\right)}_{\text{"}0\text{"}}$$

$$\iint_S xy\, dx\,dy = 0$$ perché $f(x,y) = xy$ verifica $f(x,-y) = -f(x,y)$ e S è simmetrico risp. asse x

$$\iint_T xy\,dx\,dy = \int_0^1 dx \int_{x-1}^1 dy\; xy =$$

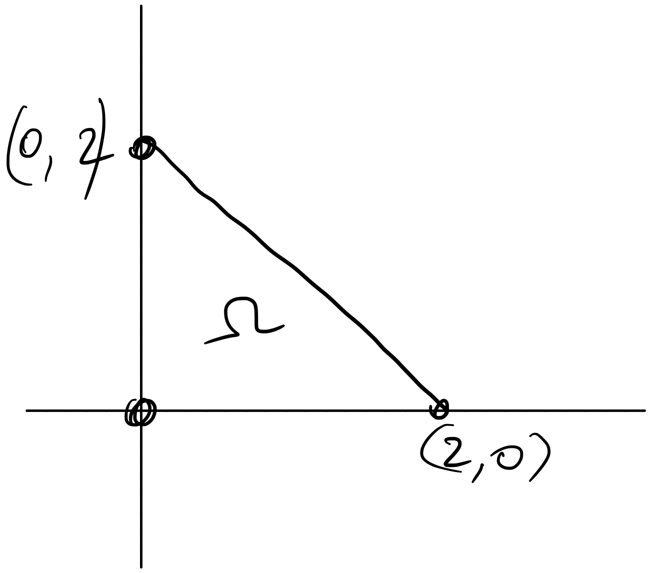
$$= \frac{1}{2}\int_0^1 dx\, x\left[1-(x-1)^2\right] = \frac{1}{2}\int_0^1 dx\; x\left[\cancel{1}-x^2-\cancel{1}+2x\right] =$$

$$= \frac{1}{2}\int_0^1 dx\,\left(-x^3+2x^2\right) = \frac{1}{2}\left(-\frac{1}{4}+\frac{2}{3}\right) =$$

$$= \frac{-3+8}{24} = \frac{5}{24}$$

$$\iint_\Omega (x-2y)\,dx\,dy$$

Ω è il triangolo di vertici $(0,0)$, $(0,2)$, $(2,0)$



$$\iint_\Omega (x-2y)\,dx\,dy = \int_0^2 dx \left(\int_0^{2-x} dy\,(x-2y) \right) =$$

$$= \int_0^2 dx \left[x(2-x) - y^2 \Big|_{y=0}^{y=2-x} \right] =$$

$$= \int_0^2 dx \left(2x - x^2 - (2-x)^2 \right) =$$

$$= \int_0^2 dx \left(2x - x^2 - 4 - x^2 + 4x \right)$$

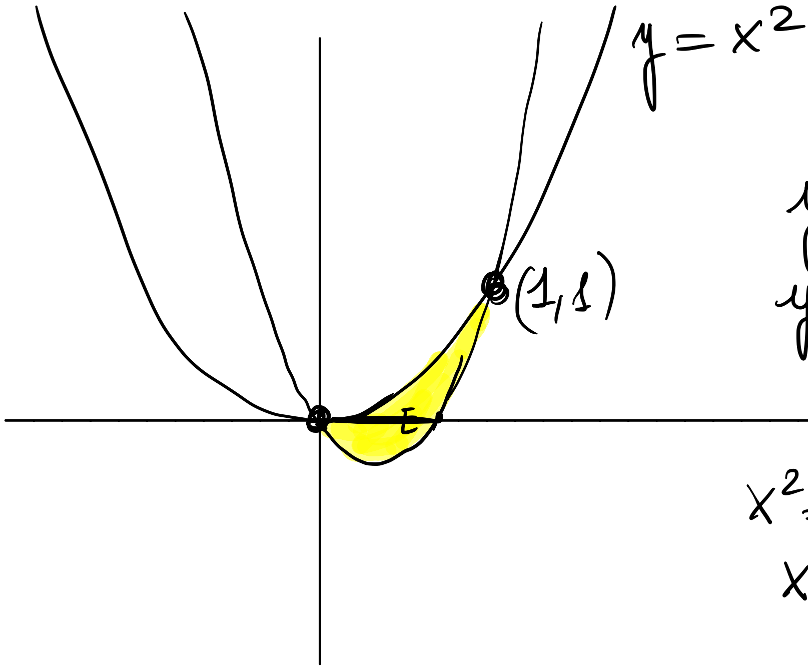
Disegnare l'insieme

$$E = \{(x,y) \in \mathbb{R}^2 : \quad 2x^2 - x \leq y \leq x^2\}$$

e scrivere una formula di riduzione per calcolare

$$\iint_E f(x,y)\,dx\,dy \qquad \forall\, f(x,y) \text{ continua.}$$

Successivamente, invertire l'ordine di integrazione.



$$y = x^2$$

$$y = 2x^2 - x$$

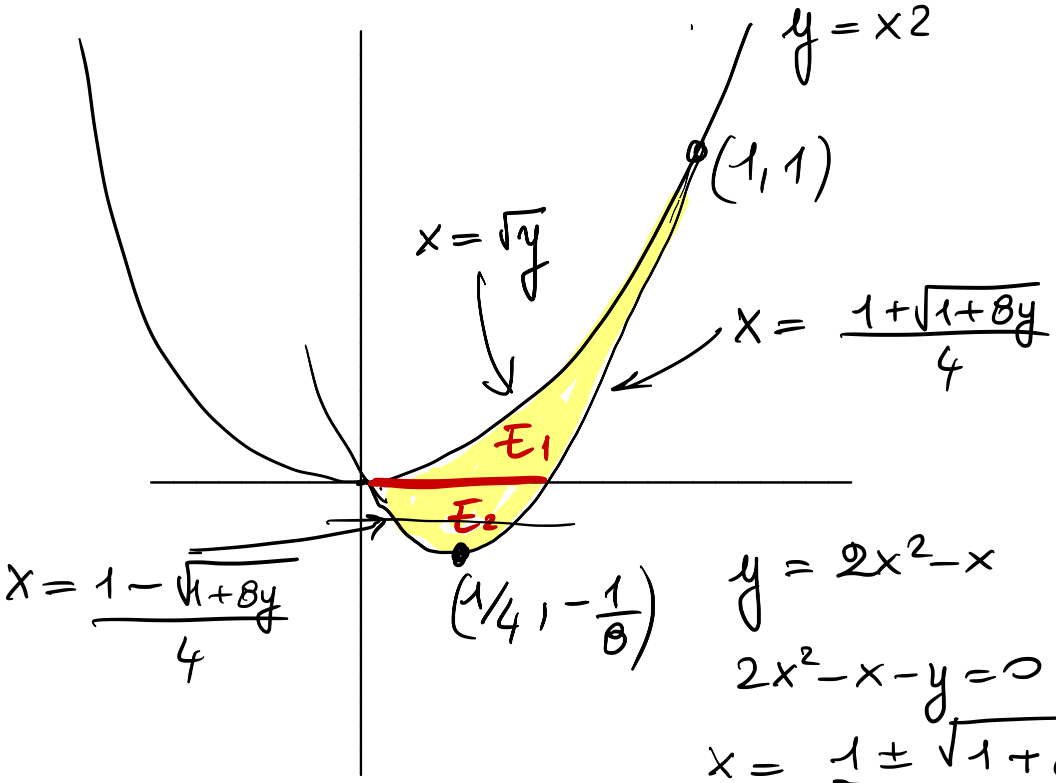
$$x^2 = 2x^2 - x$$

$$x^2 - x = 0$$

$$x = 0$$

$$x = 1$$

$$\iint_E f(x,y)\,dx\,dy = \int_0^1 dx \left(\int_{2x^2-x}^{x^2} dy \; f(x,y) \right)$$



$$2x^2 - x - y = 0$$

$$x = \frac{1 \pm \sqrt{1+8y}}{4}$$

$$\iint_E f(x,y)\,dx\,dy = \iint_{E_1} \cdots + \iint_{E_2} \cdots =$$

$$= \int_0^1 dy \left(\int_{\sqrt{y}}^{\frac{1+\sqrt{1+8y}}{4}} dx \; f(x,y) \right) + \int_{-1/8}^{0} dy \left(\int_{\frac{1-\sqrt{1+8y}}{4}}^{\frac{1+\sqrt{1+8y}}{4}} dx \; f(x,y) \right)$$

Disegnare la curva

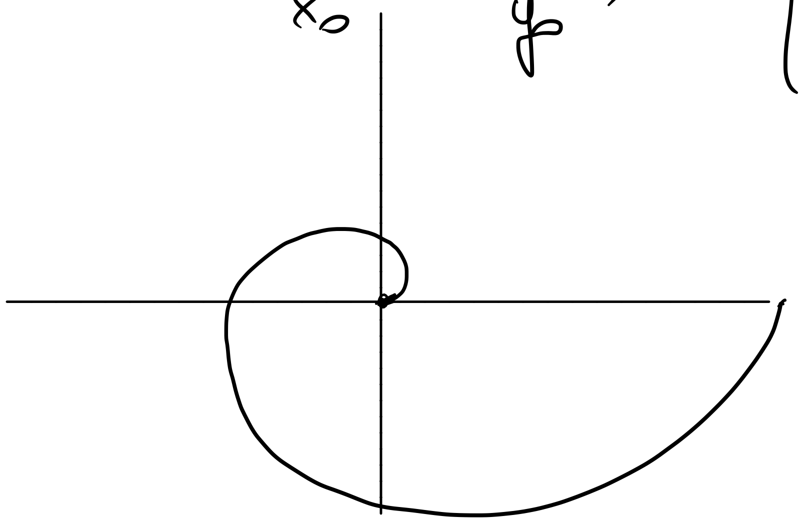
$$\underline{\gamma}(t) = (t^2 \cos t \ , \ t^2 \operatorname{sen} t) \qquad t \in [0, 2\pi]$$

e calcolarne versore tg. e retta tg nel

pto $\quad P\left(\dfrac{\pi^2}{18}, \ \dfrac{\pi^2\sqrt{3}}{18}\right)$

$\qquad\quad x_0 \qquad\quad y_0$

$$\begin{cases} t^2 \cos t = \dfrac{\pi^2}{18} \\ \\ t^2 \operatorname{sen} t = \dfrac{\pi^2\sqrt{3}}{18} \end{cases}$$



$\dfrac{\pi^2}{9} \ \dfrac{1}{2} \overset{?}{=} \dfrac{\pi^2}{18}$ ok

$\dfrac{\pi^2}{9} \ \dfrac{\sqrt{3}}{2} \overset{?}{=} \dfrac{\pi^2}{18}\sqrt{3}$

Ok.

$$\operatorname{tg} t = \frac{\operatorname{sen} t}{\cos t} = \sqrt{3}$$

$t = \dfrac{\pi}{3}$ opp. $\cancel{\dfrac{4}{3}\pi}$

$$\boxed{\text{Ok} \quad t = \frac{\pi}{3}}$$

$$\underline{\gamma}(t) = \left(t^2 \cos t \, , \, t^2 \operatorname{sen} t\right)$$

$$\underline{\gamma}'(t) = \left(2t \cos t - t^2 \operatorname{sen} t \, , \, 2t \operatorname{sen} t + t^2 \cos t\right)$$

$$\underline{\gamma}'\left(\frac{\pi}{3}\right) = \left(\frac{2\pi}{3}\frac{1}{2} - \frac{\pi^2}{9}\frac{\sqrt{3}}{2} \, , \, \frac{2\pi}{3}\frac{\sqrt{3}}{2} + \frac{\pi^2}{18}\right) = (V_1, V_2)$$

$$\underline{T}\left(\frac{\pi}{3}\right) = \frac{\left(\frac{\pi}{3} - \frac{\pi^2\sqrt{3}}{18} \, , \, \frac{\pi}{3}\sqrt{3} + \frac{\pi^2}{18}\right)}{\sqrt{\left(\frac{\pi}{3} - \frac{\pi^2\sqrt{3}}{18}\right)^2 + \left(\frac{\pi}{3}\sqrt{3} + \frac{\pi^2}{18}\right)^2}}$$

Retta tg.

$$\frac{x - x_0}{V_1} = \frac{y - y_0}{V_2}$$

$$(x, y) = (x_0, y_0) + t\,(V_1, V_2)$$

$$\begin{cases} x = x_0 + t\,V_1 \\ y = y_0 + t\,V_2 \end{cases} \qquad t \in \mathbb{R}$$

noti

Trovare tutti gli $(a,b)\in\mathbb{R}^2$ che rendono conservativo il campo

$$\underline{F}(x,y) = \left(\frac{ay}{1+x^2y^2} + \frac{2x}{1+x^2},\ \frac{x}{1+x^2y^2} - b\cos y\right)$$

dominio: $\mathbb{R}^2$: conservativo $\Leftrightarrow$ irrotaz.

$$\frac{\partial}{\partial y}\left(\frac{ay}{1+x^2y^2} + \cancel{\frac{2x}{1+x^2}}\right) \overset{?}{=} \frac{\partial}{\partial x}\left(\frac{x}{1+x^2y^2} - \cancel{b\cos y}\right)$$

$$a\,\frac{1+x^2y^2 - y\cdot 2x^2y}{\cancel{(1+x^2y^2)^2}} \overset{?}{=} \frac{1+x^2y^2 - 2x^2y^2}{\cancel{(1+x^2y^2)^2}}$$

$$a(1-x^2y^2) \overset{?}{=} 1-x^2y^2$$

$$\boxed{\begin{array}{l} a=1 \\ b \text{ qualsiasi} \end{array}}$$

$$V_x(x,y) = \frac{y}{1+x^2y^2} + \frac{2x}{1+x^2}$$

$$V(x,y) = \int\left(\frac{y}{1+x^2y^2} + \frac{2x}{1+x^2}\right)dx =$$

$$= \operatorname{arctg}(xy) + \ln(1+x^2) + g(y)$$

$$V(x,y) = \int \left(\frac{y}{1+x^2y^2} + \frac{2x}{1+x^2} \right) dx =$$

$$= \operatorname{arctg}(xy) + \ln(1+x^2) + g(y)$$

$$V_y(x,y) = \cancel{\frac{x}{1+x^2y^2}} - b\cos y$$

$$\shortparallel$$

$$\cancel{\frac{x}{1+x^2y^2}} + g'(y)$$

$$g'(y) = -b\cos y \Rightarrow g(y) = -b\,\text{sen}\,y + c$$

$$V(x,y) = \operatorname{arctg}(xy) + \ln(1+x^2) - b\,\text{sen}\,y$$

Dire per quali α il campo vett.

$$\underline{F}(x,y) = (\alpha x^2 y^2,\ 2x^3 y - 3y^2)$$

è conservativo in $\mathbb{R}^2$?

$$\frac{\partial}{\partial y}(\alpha x^2 y^2) \overset{?}{=} \frac{\partial}{\partial x}(2x^3 y - 3y^2)$$

$$2\alpha x^2 y \overset{?}{=} 6x^2 y$$

ok se $\quad 2\alpha = 6 \qquad \alpha = 3$

Dire per quali α il campo vett.

$$\underline{F}(x,y) = \left(\alpha x^2 y^2,\quad 2x^2 y - 3y^2\right)$$

è conservativo in $\mathbb{R}^2$.

$$2\alpha x^2 y \overset{?}{\equiv} 4xy$$

Per nessun valore di α

Il lavoro del campo

$$\underline{F}(x,y) = \left(2y + x^2 ,\ 4xy + 1\right)$$

per spostare una particella da $(0,0)$ a $(1,1)$

vale ??

$$\frac{\partial}{\partial y}\left(2y + x^2\right) \overset{?}{=} \frac{\partial}{\partial x}\left(4xy + 1\right)$$

$$2 \overset{?}{=} 4y \qquad \underline{\text{NO}}$$

Il lavoro del campo

$$\underline{F}(x,y) = \left(2y^2 + x^2,\ 4xy + 1\right)$$

per spostare una particella da $(0,0)$ a $(1,1)$ vale ??

$$\frac{\partial}{\partial y}\left(2y^2 + x^2\right) \overset{?}{\equiv} \frac{\partial}{\partial x}\left(4xy + 1\right)$$

$$4y \overset{?}{\equiv} 4y.$$

$$V(x,y) = \int \left(2y^2 + x^2\right) dx = 2xy^2 + \frac{x^3}{3} + g(y)$$

$$V_y(x,y) = \cancel{4xy} + 1$$

$$\shortparallel$$

$$\cancel{4xy} + g'(y) \Rightarrow g'(y) = 1 \Rightarrow g(y) = y + c.$$

$$V(x,y) = 2xy^2 + \frac{x^3}{3} + y$$

$$\text{Lavoro} = V(1,1) - \cancel{V(0,0)} = 2 + \frac{1}{3} + 1 = \frac{10}{3}$$