Sia data una curva γ in coord. polari della forma

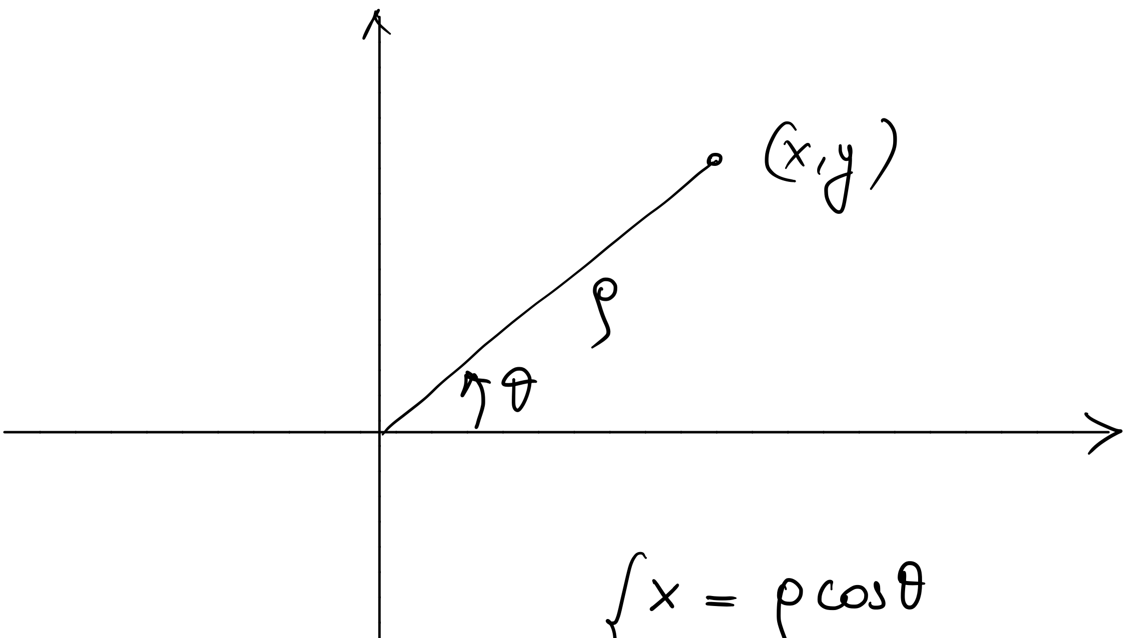
$$\rho = \rho(\theta) \qquad \theta \in [a, b].$$

cioè di eq^ni parametriche

$$\begin{cases} x(\theta) = \rho(\theta)\cos\theta \\ y(\theta) = \rho(\theta)\,\mathrm{sen}\,\theta \end{cases}$$

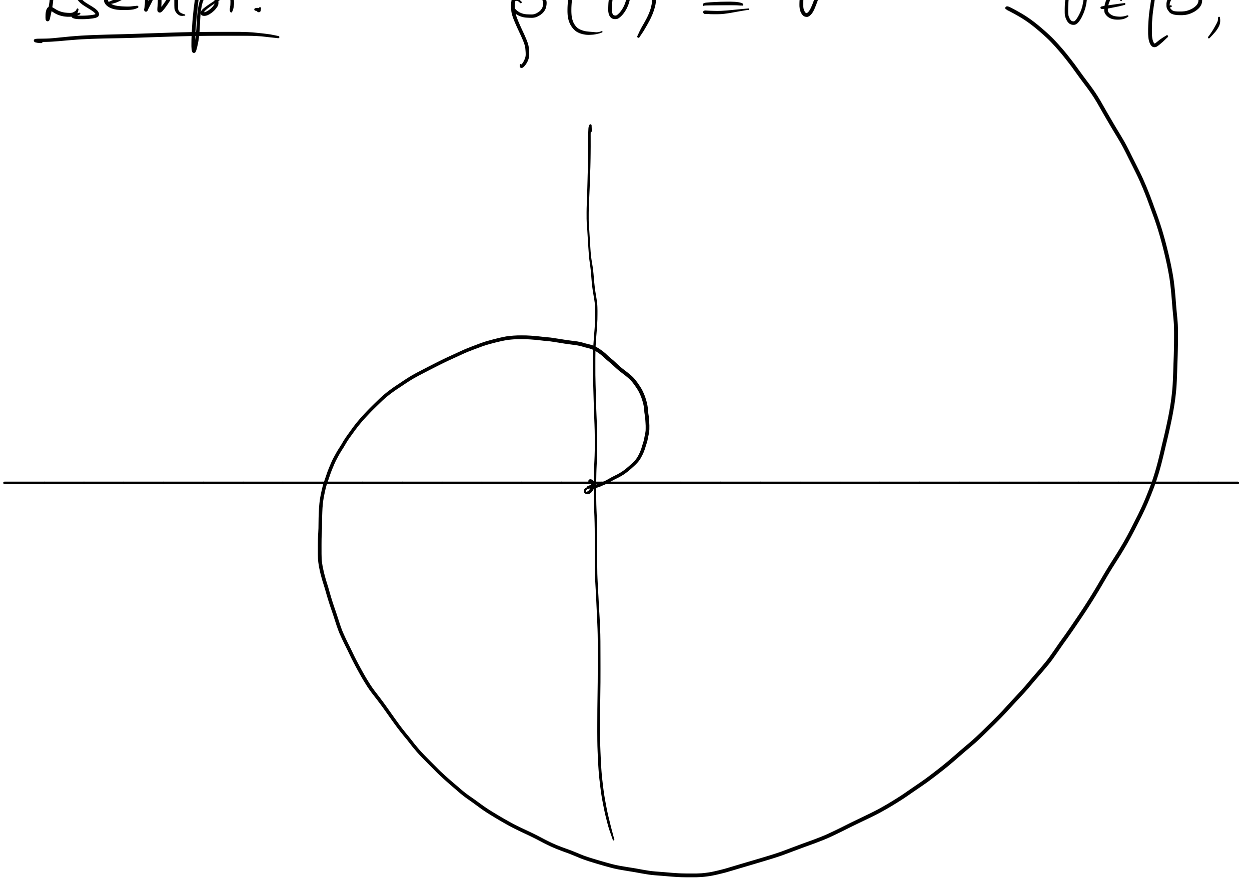
Mostrare che la lunghezza della curva è data da:

$$L(\gamma) = \int_a^b \sqrt{\rho(\theta)^2 + (\rho'(\theta))^2}\, d\theta$$



$$\begin{cases} x = \rho\cos\theta \\ y = \rho\,\mathrm{sen}\,\theta \end{cases}$$

Esempi: $\rho(\theta) = \theta \qquad \theta \in [0, b]$



$\rho(\theta) = e^{\theta}$ spirale logaritmica.

$$L(\gamma) = \int_a^b \sqrt{x'(\theta)^2 + y'(\theta)^2}\, d\theta = (*)$$

$$x(\theta) = \rho(\theta)\cos\theta \Rightarrow x'(\theta) = \rho'(\theta)\cos\theta - \rho(\theta)\operatorname{sen}\theta$$

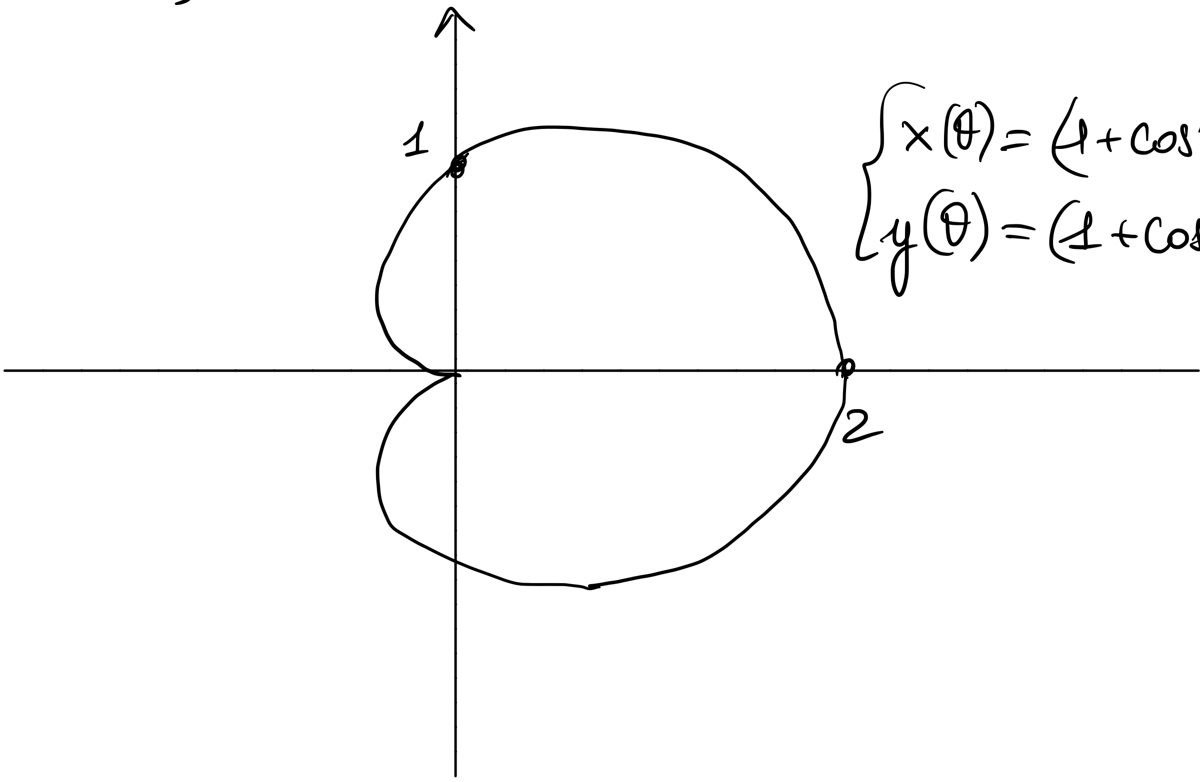
$$y(\theta) = \rho(\theta)\operatorname{sen}\theta \Rightarrow y'(\theta) = \rho'(\theta)\operatorname{sen}\theta + \rho(\theta)\cos\theta.$$

$$x'(\theta)^2 + y'(\theta)^2 = \rho'(\theta)^2 + \rho(\theta)^2$$

$$(*) = \int_a^b \sqrt{\rho(\theta)^2 + \rho'(\theta)^2}\, d\theta.$$

Calcolare la lunghezza della cardioide

$$\rho = 1 + \cos\theta \qquad -\pi \leq \theta \leq \pi.$$



$$\begin{cases} x(\theta) = (1+\cos\theta)\cos\theta \\ y(\theta) = (1+\cos\theta)\,\mathrm{sen}\,\theta \end{cases}$$

$$L(\gamma) = \int_{-\pi}^{\pi} \sqrt{\rho(\theta)^2 + \rho'(\theta)^2}\, d\theta =$$

$$= \int_{-\pi}^{\pi} \sqrt{(1+\cos\theta)^2 + \mathrm{sen}^2\theta}\, d\theta =$$

$$= \int_{-\pi}^{\pi} \sqrt{2 + 2\cos\theta}\, d\theta =$$

$$= \sqrt{2} \int_{-\pi}^{\pi} \sqrt{1+\cos\theta}\, d\theta = 2\sqrt{2} \int_{0}^{\pi} \sqrt{1+\cos\theta}\, d\theta$$

$$2\sqrt{2}\int_0^{\pi}\sqrt{1+\cos\theta}\,d\theta = 2\sqrt{2}\cdot\sqrt{2}\int_0^{\pi}\cos\frac{\theta}{2}\,d\theta = (*)$$

$$\frac{1+\cos\theta}{2} = \cos^2\left(\frac{\theta}{2}\right) \Rightarrow \sqrt{1+\cos\theta} = \sqrt{2}\sqrt{\cos^2\left(\frac{\theta}{2}\right)} =$$

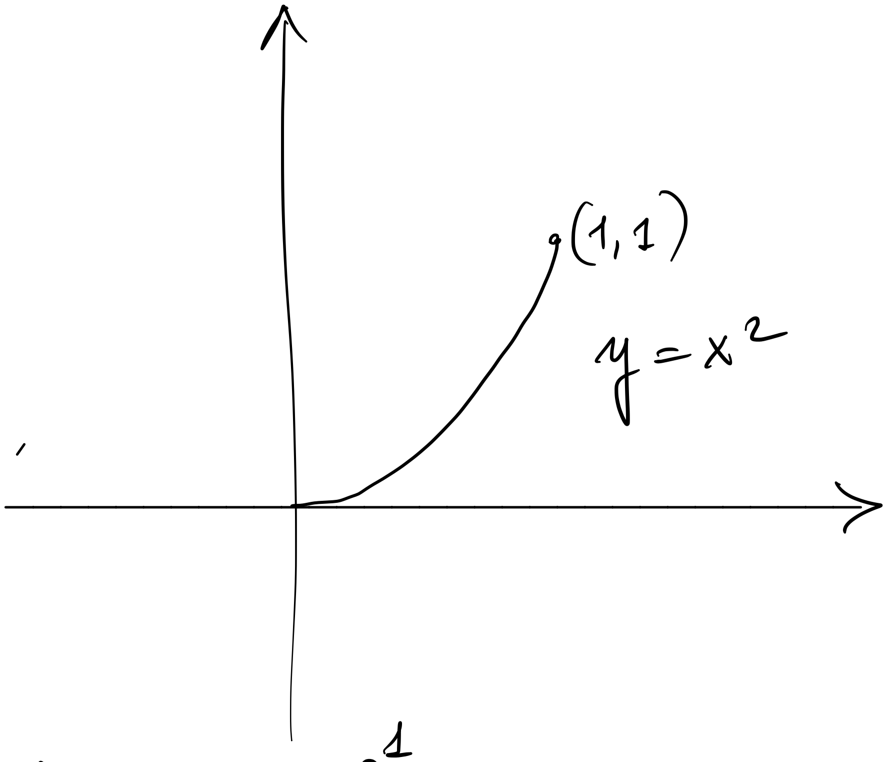
$$= \sqrt{2}\left|\cos\frac{\theta}{2}\right| =$$

$\theta\in[0,\pi]$

$$= \sqrt{2}\cos\frac{\theta}{2}$$

$$(*) = 4\cdot 2\ \operatorname{sen}\left(\frac{\theta}{2}\right)\Big|_0^{\pi} = 8$$

Detto γ l'arco di parabola $y = x^2$, con $x \in [0,1]$, calcolare $\int_\gamma x \, ds$



$$\begin{cases} x = t \\ y = t^2 \end{cases} \qquad t \in [0,1]$$

$$\int_\gamma x \, ds = \int_0^1 t \sqrt{1+4t^2} \, dt =$$

$$= \frac{1}{8} \int_0^1 8t \sqrt{1+4t^2} \, dt \qquad \begin{bmatrix} 1+4t^2 = v \\ 8t\,dt = dv \end{bmatrix}$$

$$= \frac{1}{8} \int_1^5 \sqrt{v} \, dv = \frac{1}{8} \cdot \frac{2}{3} \left. v^{3/2} \right|_1^5 =$$

$$= \frac{1}{12} \left(5\sqrt{5} - 1 \right)$$

Il lavoro del campo vett.

$\underline{F}(x,y) = (2y + x^2, 4xy + 1)$

per spostare una particella dal pto $(0,0)$ al pto $(1,1)$ vale:

$\boxed{A}$ -1

$\boxed{B}$ 1

$\boxed{C}$ 3

$\boxed{D}$ -3

$\boxed{E}$ dipende dal percorso seguito

$\boxed{F}$ nessuna delle preced.

Vediamo se è irrotaz.

$$(2y + x^2)_y \overset{?}{=} (4xy + 1)_x$$

$$2 \overset{?}{=} 4y \qquad \text{no,}$$

non è irrotazionale $\Rightarrow$ non è conservativo $\Rightarrow$

$\Rightarrow$ dipende dal percorso seguito.

Dire per quali valori $\alpha \in \mathbb{R}$ il campo

$$\underline{F}(x,y) = (\alpha x^2 y^2 \ , \ 2x^3 y - 3y^2)$$

è conservativo in $\mathbb{R}^2$

[A] per $\alpha = -1$

[B] per $\alpha = 2$

~~[C]~~ per $\alpha = 3$

[D] per nessun α

[E] nessuna delle risp. preced.

$\mathbb{R}^2$ sempl. connesso $\Rightarrow$ [conservativo $\Leftrightarrow$ irrotazionale]

$$(\alpha x^2 y^2)_y \overset{?}{=} (2x^3 y - 3y^2)_x \quad \text{in } \mathbb{R}^2$$

$$2\alpha x^2 y \overset{?}{=} 6x^2 y$$

$$\boxed{\alpha = 3}$$

Si consideri la spirale γ

$$\begin{cases} x(t) = t\cos t \\ y(t) = t\,\mathrm{sen}\,t \end{cases}$$

Il versore tg a γ nel pto $(2\pi, 0)$ vale...?

$(2\pi, 0)$ corrisponde a $t = 2\pi$

$$\underline{\gamma}'(t) = (\cos t - t\,\mathrm{sen}\,t,\ \mathrm{sen}\,t + t\cos t)$$

$$\underline{\gamma}'(2\pi) = (1,\ 2\pi).$$

$$\underline{T}(2\pi) = \frac{\underline{\gamma}'(2\pi)}{|\underline{\gamma}'(2\pi)|} = \frac{(1,\ 2\pi)}{\sqrt{1+4\pi^2}} = \left(\frac{1}{\sqrt{1+4\pi^2}},\ \frac{2\pi}{\sqrt{1+4\pi^2}}\right)$$

Il campo
$$\underline{F}(x,y) = \left(\sin(xy) + xy\cos(xy),\ x^2\cos(xy)\right)$$
è, nel suo dominio

~~A~~ irrotaz. e conservativo

B irrotaz. ma non conservativo

C conservativo ma non irrotazionale.

D né conservativo né irrotazionale

E i dati non sono suff.ti a concludere.

Dom. $\mathbb{R}^2$

$$\left(\sin(xy) + xy\cos(xy)\right)_y \overset{?}{=} \left(x^2\cos(xy)\right)_x$$

$$x\cos(xy) + x\cos(xy) - x^2y\ \text{sen}(xy) \overset{?}{=}$$

$$\overset{?}{=} 2x\cos(xy) - yx^2\ \text{sen}(xy)$$

sì.

irrotaz. $\underset{\mathbb{R}^2}{\Longrightarrow}$ conservativo.