2. $f(x,y) = \sqrt{3+x^4+2y^4}$ Dominio: $\mathbb{R}^2$.

$$f_x(x,y) = \frac{2\cancel{4}x^3}{\cancel{2}\sqrt{3+x^4+2y^4}}; \quad f_y(x,y) = \frac{4\cancel{8}y^3}{\cancel{2}\sqrt{3+x^4+2y^4}}$$

$\forall (x,y) \in \mathbb{R}^2$.

3. $f(x,y) = \sqrt{x^4 + 2y^4}$

dominio: $\mathbb{R}^2$

$$f_x(x,y) = \frac{2x^3}{\sqrt{x^4+2y^4}} \; ; \; f_y(x,y) = \frac{4y^3}{\sqrt{x^4+2y^4}}$$

$$\forall (x,y) \in \mathbb{R}^2 \setminus \{(0,0)\}$$

Cosa succede in $(0,0)$?

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h\to 0} \frac{f(h,0) - \cancel{f(0,0)}}{h} =$$

$$= \lim_{h\to 0} \frac{\sqrt{h^4}}{h} = \lim_{h\to 0} \frac{h^{\cancel{2}}}{\cancel{h}} = 0$$

Nella stessa maniera si ottiene

$$\frac{\partial f}{\partial y}(0,0) = 0$$

3! $f(x,y) = \sqrt{x^2 + 2y^4}$

ovviamente è derivabile parzialmente in $\mathbb{R}^2 \setminus \{(0,0)\}$ (scrivere le formule)

$$\frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{\sqrt{h^2} - 0}{h} = \lim_{h \to 0} \frac{|h|}{h} \nexists$$

$$\frac{\partial f}{\partial y}(0,0) = \; . \; . \; . \; . \; . \; = 0$$

Calcolare, se esistono, le derivate direzionali $\frac{\partial f}{\partial \underline{v}}(x_0, y_0)$ delle funzioni f indicate, nei punti (x_0, y_0) indicati, lungo le direzioni $\underline{v}$ individuate dai vettori $\underline{w}$ indicati.
(N.B.: $\underline{w}$ potrebbe non essere una direzione).

7. $f(x,y) = x^3 y - 3y^2$, $(x_0, y_0) = (1, -2)$, $\underline{w} = (-5, 12)$

$$\underline{v} = \frac{(-5, 12)}{\sqrt{5^2 + 12^2}} = \left(-\frac{5}{13}, \frac{12}{13}\right).$$

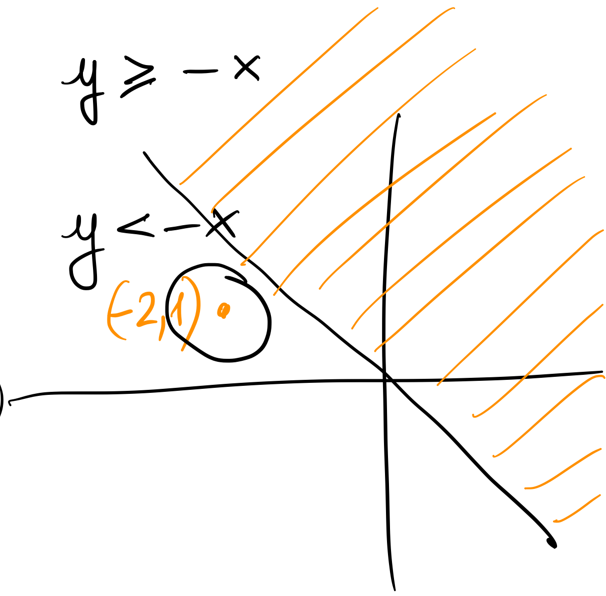
$$f_x(x,y) = 3x^2 y \; ; \quad f_y(x,y) = x^3 - 6y; \quad \nabla f(1,-2) = (-6, 13)$$

$$\frac{\partial f}{\partial \underline{v}}(1,-2) = \nabla f(1,-2) \cdot \underline{v} = (-6, 13) \cdot \left(-\frac{5}{13}, \frac{12}{13}\right) =$$

$$= \frac{30}{13} + 12 = \frac{186}{13}$$

9. $f(x,y) = |x+y|$ $\qquad (x_0, y_0) = (-2, 1)$, $\underline{w} = (2, 3)$

$$f(x,y) = |x+y| = \begin{cases} x+y & \text{se } y \geq -x \\ -x-y & \phantom{\text{se }} y < -x \end{cases}$$



OSS in un intorno di $(-2,1)$ si ha
$f(x,y) = -x-y$ (di classe C^1)

$$\underline{v} = \left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right)$$

$$\frac{\partial f}{\partial \underline{v}}(-2,1) = \nabla f(-2,1) \cdot \underline{v} = (-1,-1) \cdot \left(\frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}}\right) = -\frac{5}{\sqrt{13}}$$

18. $f(x,y) = 2x^3 + 6xy^2 + 3y^3 - 150x$

$f_x(x,y) = 6x^2 + 6y^2 - 150$ $\qquad$ $f_{xx}(x,y) = 12x$

$f_y(x,y) = 12xy + 9y^2$ $\qquad$ $f_{xy}(x,y) = 12y$

$f_{yy}(x,y) = 12x + 18y.$

Pti critici:
$$\begin{cases} x^2 + y^2 - 25 = 0 \\ y(4x+3y) = 0 \end{cases}$$

$$\begin{cases} y = 0 \\ x^2 - 25 = 0 \end{cases} \qquad \begin{cases} y = -\frac{4}{3}x \\ x^2 + \frac{16}{9}x^2 = 25 \end{cases}$$

$(\pm 5, 0)$

$\frac{25}{9}x^2 = 25$

$x = \pm 3 \qquad y = \mp 4$

Pti critici: $(\pm 5, 0)$, $(3,-4)$, $(-3,4)$

$$D^2 f(x,y) = \begin{bmatrix} 12x & 12y \\ 12y & 12x + 18y \end{bmatrix}$$

$$D^2 f(5,0) = \begin{bmatrix} 60 & 0 \\ 0 & 60 \end{bmatrix} \Rightarrow (5,0) \text{ è pto di min. rel. stretto}$$

$$D^2 f(-5,0) = \begin{bmatrix} -60 & 0 \\ 0 & -60 \end{bmatrix} \Rightarrow (-5,0) \text{ è pto di max. rel. stretto.}$$

$$D^2 f(x,y) = \begin{bmatrix} 12x & 12y \\ 12y & 12x + 18y \end{bmatrix}$$

$$D^2 f(3,-4) = \begin{bmatrix} 36 & -48 \\ -48 & -36 \end{bmatrix}.$$

$$\det D^2 f = -36^2 - 48^2 < 0 \Rightarrow (3,-4) \text{ sella.}$$

né max né min. rel.

$$D^2 f(-3,4) = \begin{bmatrix} -36 & 48 \\ 48 & 36 \end{bmatrix}$$

$$\det D^2 f < 0 \Rightarrow (-4,3) \text{ sella.}$$

$$f(x,y) = x^4 - 2xy + 2y^2$$

1) trovare e classificare i pti critici

2) Trovare, se esistono, max. e min. assoluti di f nel triangolo chiuso di vertici $(0,0), (1,0), (1,1)$.

$$f_x(x,y) = 4x^3 - 2y \qquad f_{xx}(x,y) = 12x^2$$

$$f_y(x,y) = -2x + 4y \qquad f_{xy}(x,y) = -2$$

$$f_{yy}(x,y) = 4$$

Pti critici

$$\begin{cases} 2x^3 - y = 0 \\ -x + 2y = 0 \end{cases}$$

$$x = 2y$$

$$16y^3 - y = 0 \quad \begin{cases} y = 0 \\ y = \pm \frac{1}{4} \end{cases}$$

$$y(16y^2 - 1)$$

$$(0,0) \quad \left(\frac{1}{2}, \frac{1}{4}\right) \quad \left(-\frac{1}{2}, -\frac{1}{4}\right)$$

$$D^2 f(x,y) = \begin{bmatrix} 12x^2 & -2 \\ -2 & 4 \end{bmatrix}$$

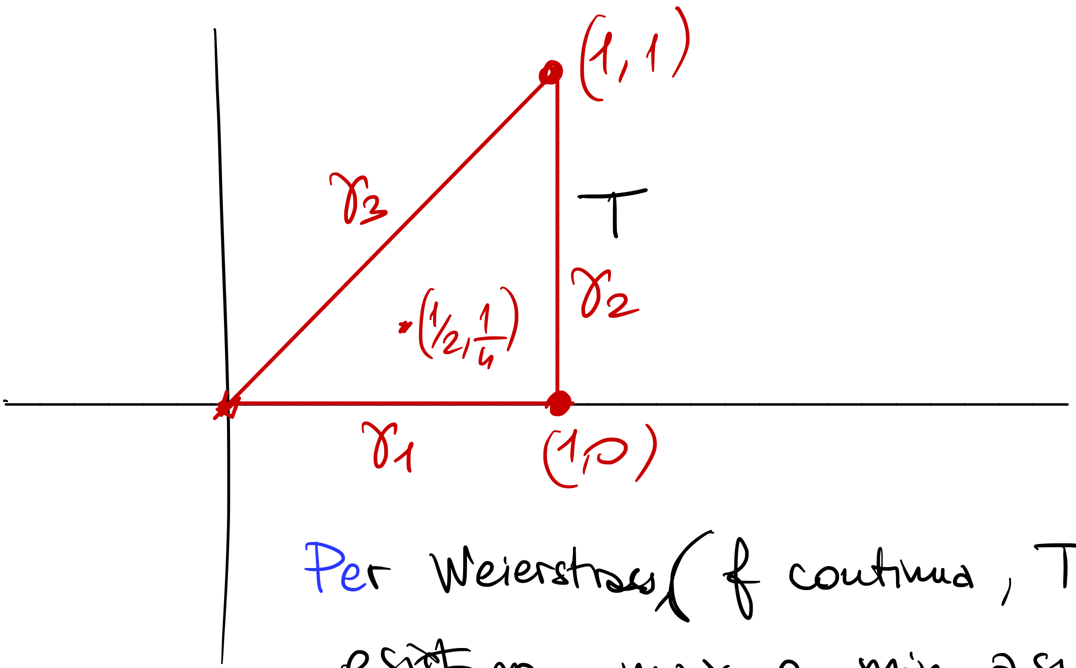
$$D^2 f(0,0) = \begin{bmatrix} 0 & -2 \\ -2 & 4 \end{bmatrix} \Rightarrow \det D^2 f = -4 < 0$$

$(0,0)$ sella.

$$D^2 f\left(\pm\frac{1}{2}, \pm\frac{1}{4}\right) = \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \Rightarrow \det D^2 f = 8 > 0$$

$$f_{xx} = 3 > 0$$

$\Rightarrow \left(\pm\frac{1}{2}, \pm\frac{1}{4}\right)$ pti di min. rel. stretto.



Per Weierstrass (f continua , T chiuso e limitato) esistono max. e min. assoluti.

Possono essere assunti: 1) nei pti critici interni : solo $\left(\frac{1}{2}, \frac{1}{4}\right)$

2) nei pti di non derivabilità: nessuno.

3) su ∂T

γ_1 $\quad \varphi_1(x) := f(x,0) = x^4 \qquad x \in [0,1]$

$(0,0) \quad (1,0)$

$y \in [0,1]$.

γ_2 $\quad \varphi_2(y) = f(1,y) = 1 - 2y + 2y^2 \qquad \varphi_2'(y) = -2 + 4y = 0$

$y = 1/2$.

$(1,0) \quad \left(1, \frac{1}{2}\right) \quad (1,1)$

γ_3 $\quad \varphi_3(x) = f(x,x) = x^4 \qquad x \in [0,1]$

$(0,0), \quad (1,1).$

$$f(x,y) = x^4 - 2xy + 2y^2$$

Pti da controllare: $(0,0)$, $(1,0)$, $(1,1)$, $\left(\frac{1}{2}, \frac{1}{4}\right)$, $\left(1, \frac{1}{2}\right)$

$$f(0,0) = 0$$

$$f(1,0) = 1 = f(1,1)$$

$$f\left(\frac{1}{2}, \frac{1}{4}\right) = \frac{1}{16} - \frac{2}{8} + \frac{2}{16} = \frac{1-4+2}{16} = -\frac{1}{16}$$

$$f\left(1, \frac{1}{2}\right) = 1 - 1 + \frac{2}{4} = \frac{1}{2}.$$

$$\min_{(x,y)\in T} f(x,y) = f\left(\frac{1}{2}, \frac{1}{4}\right) = -\frac{1}{16}$$

$$\max_{(x,y)\in T} f(x,y) = f(1,0) = f(1,1) = 1$$