

Esercizi settimana 1: successioni e limiti

Tutoraggio di Analisi Matematica (a.a. 2017/18)

- Calcolare i seguenti limiti:

$$1. \lim_{n \rightarrow +\infty} \frac{n^{-3/4} + \left(\frac{1}{n}\right)^{6/5} - \frac{1}{n^{10}}}{\frac{1}{\sqrt{n}} - n^{-7/4}} \quad [0]$$

$$2. \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{2n}\right)^{3n} \quad [e^{3/2}]$$

$$3. \lim_{n \rightarrow +\infty} \frac{3^{-n} - \left(\frac{1}{2}\right)^n + \left(\frac{5}{6}\right)^n}{\left(\frac{3}{4}\right)^n - \left(\frac{2}{5}\right)^n} \quad [+\infty]$$

$$4. \lim_{n \rightarrow +\infty} \frac{\log\left(1 + \frac{3}{n^2}\right)}{\sin\left(\frac{2}{n^2}\right)} \quad [3/2]$$

$$5. \lim_{n \rightarrow +\infty} \frac{\left(\frac{4}{3}\right)^{-n} - \left(\frac{5}{7}\right)^n}{\left(\frac{2}{3}\right)^n + \left(\frac{6}{5}\right)^{-n} - \left(\frac{4}{7}\right)^n} \quad [0]$$

$$6. \lim_{n \rightarrow +\infty} \frac{e^{n^2} + n^5}{n^n} \quad [+\infty]$$

$$7. \lim_{n \rightarrow +\infty} \frac{n^{-6/7} - e^{-n} + \left(\frac{1}{2}\right)^n}{\left(\frac{4}{3}\right)^{-2n} + \frac{1}{n^4}} \quad [+\infty]$$

$$8. \lim_{n \rightarrow +\infty} \frac{\log\left(\frac{1}{n^5}\right) + \log\sqrt{n}}{2\log(n^6 + n^2)} \quad [-3/8]$$

$$9. \lim_{n \rightarrow +\infty} \left(n - \frac{4}{n}\right) \left(\sqrt{1 + \frac{3}{n}} - 1\right) \quad [3/2]$$

[Suggerimento: ricordare che $\frac{(1+\varepsilon_n)^\nu - 1}{\varepsilon_n} \rightarrow \nu$ per $\varepsilon_n \rightarrow 0, \forall \nu \in \mathbb{R}$.]

$$10. \lim_{n \rightarrow +\infty} \frac{1}{\sqrt[n]{2^{2^n}}} \left[n \log \left(\frac{n+3}{n} \right)^2 \right] \quad [0]$$

$$11. \lim_{n \rightarrow +\infty} [\sin(\sin^4 n)]^{n/4} \quad [0]$$

$$12. \lim_{n \rightarrow +\infty} \left(1 + \sin \frac{1}{n^3}\right)^{n^3} \quad [e]$$

$$13. \lim_{n \rightarrow +\infty} \left(\frac{n! + 5}{n!}\right)^{n!} \quad [e^5]$$

$$14. \lim_{n \rightarrow +\infty} \left(1 - \cos \frac{1}{n}\right)^{\tan(1/n)} \quad [1]$$

$$15. \lim_{n \rightarrow +\infty} [\sin(|\sin n|^{1/4})]^{4n} \quad [0]$$

$$16. \lim_{n \rightarrow +\infty} \frac{\sin \frac{1}{n^2} - \log \left(1 + \frac{1}{2n^2+n}\right)}{1 - \cos \frac{1}{2n+1}} \quad [4]$$

$$17. \lim_{n \rightarrow +\infty} \left(1 + \frac{1}{n}\right)^{\frac{n \log n}{1+\sqrt{n}}} \quad [1]$$

$$18. \lim_{n \rightarrow +\infty} (n^4 + 1) \left(1 - \cos \frac{3}{n^2}\right) \quad [9/2]$$

$$19. \lim_{n \rightarrow +\infty} \frac{e^n - 2^{n \log n}}{n^n} \quad [0]$$

- Calcolare, al variare di $\alpha \in \mathbb{R}$,

$$\lim_{n \rightarrow +\infty} \frac{\sin \left(\frac{1}{n^{\alpha^2+1}}\right) - \log \left(1 + \frac{2}{n}\right)}{n^{2-\alpha}}.$$

[Soluzione: il limite vale 0 per $\alpha < 3$, -2 per $\alpha = 3$, $-\infty$ per $\alpha > 3$.]