

1) Calcolare i seguenti integrali doppi

$$\begin{aligned} \iint_D xye^y dx dy \quad D &= [0, 1] \times [0, 2]; \quad \iint_D \sin(x + 3y) dx dy \quad D = [0, \pi/3] \times [0, \pi] \\ \iint_D \frac{1+x^2}{1+y^2} dx dy \quad D &= [-1, 1] \times [0, \sqrt{3}]; \\ \iint_D y \cos(xy) dx dy \quad D &= [-\pi/4, \pi/4] \times [0, 1] \end{aligned}$$

2) Disegnare i domini a cui si riferiscono gli integrali seguenti e invertire l'ordine di integrazione

$$\int_0^4 \left[\int_{3x^2}^{12x} f(x, y) dy \right] dx \quad \int_0^2 \left[\int_{1-x^2/4}^{\sqrt{4-x^2}} f(x, y) dy \right] dx \quad \int_1^2 \left[\int_0^{\sqrt{4x-x^2}} f(x, y) dy \right] dx$$

3) Disegnare i seguenti insiemi e calcolarne l'area

$$\begin{aligned} A &= \left\{ (x, y) \in \mathbf{R}^2; x \in [-1, 1], -x^4 \leq y \leq 2x^2 \right\} \\ B &= \left\{ (x, y) \in \mathbf{R}^2; y \in [-1, 1], -y^2 \leq x \leq 4y^2 \right\} \end{aligned}$$

4) $\iint_T |xy| dx dy$ dove T è il triangolo di vertici A=(-1,0), B=(0,1), C=(1,0)

5) $\iint_T x dx dy$ dove T = $\{(x, y) \in \mathbf{R}^2; x^2 + 4y^2 \leq 4, 4y \geq x^2 - 4x + 4\}$

6) $\iint_T \left(\frac{x}{2+xy} \right)^2 dx dy$ dove T = $\{(x, y) \in \mathbf{R}^2; |y| \leq x \leq 1\}$

7) $\iint_T \log(x+1)^2 dx dy$ dove T = $\{(x, y) \in \mathbf{R}^2; 2 \leq x \leq 4, 1 \leq y \leq x\}$

8) Calcolare i seguenti integrali doppi usando un opportuno cambiamento di variabile

$$a) \iint_D (1 - x^2 - y^2)^{3/2} dx dy \quad D = \{(x, y) \in \mathbf{R}^2; y \geq 0, x^2 + y^2 \leq 1/4\}$$

$$b) \iint_D \frac{x+y}{(x-y)^2+1} dx dy \quad D = \{(x, y) \in \mathbf{R}^2; 0 \leq x+y \leq 1; 0 \leq x-y \leq 1\}$$

$$c) \iint_D xy dx dy \quad D = \{(x, y) \in \mathbf{R}^2; x \geq 0, y \geq 0, 4y \leq x^2 + y^2 \leq 4\}$$

$$d) \iint_T \frac{x^2}{y} dx dy \quad T = \{(x, y) \in \mathbf{R}^2; 1/2 \leq xy \leq 1, 2 \leq \frac{y}{x^2} \leq 3\}$$

$$e) \iint_T \frac{1}{\sqrt{x^2+y^2}} dx dy \quad T = \{(x, y) \in \mathbf{R}^2; 3 \leq x^2 + y^2 \leq 2y\}$$