SOLUZIONI SCHEMATICHE

①

Basi possibili: a) $|\ell_z\ S_z^p\ S_z^e\rangle_{LS^eS^p}$

b) $|\ell_z\ S^{tot}\ S_z^{tot}\rangle_{LS^{tot}}$

c) $|J\ J_z\ S_{tot}\rangle_J$

$$|1\ -\tfrac{1}{2}\ \tfrac{1}{2}\rangle_{LS^eS^p} = \frac{1}{2}|2\ 1\ 1\rangle_J + \frac{1}{2}|1\ 1\ 1\rangle_J - \frac{1}{\sqrt{2}}|1\ 1\ 0\rangle_J$$

$$\text{Prob}(J^2 = 6\hbar^2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\text{Prob}(J^2 = 2\hbar^2) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{3}{4}$$

②

$$\phi(0) = |n=2, \ell=1, \ell_z=1\rangle \frac{1}{\sqrt{2}}\left(|1\ 0\rangle_{S_{tot}} - |0\ 0\rangle_{S_{tot}}\right)$$

$$\phi(t) = |n=2, \ell=1, \ell_z=1\rangle \frac{1}{\sqrt{2}} e^{iE_I t/4\hbar} e^{i\frac{3\alpha t}{4\hbar}} \times$$

$$\left(e^{-i\alpha t/\hbar}|1\ 0\rangle_{S_{tot}} - |0\ 0\rangle_{S_{tot}}\right)$$

$$\langle\phi(t)|\phi(0)\rangle = \frac{1}{2}\left(1 + \exp\left(\frac{i\alpha t}{\hbar}\right)\right)$$

$$P(t) = \frac{1}{2}\left(1 + \cos\frac{\alpha t}{\hbar}\right)$$

③

Lo stato fondamentale è $\psi_f = |n=1\ \ell=0\ \ell_z=0\rangle|S_{tot}=0\ S_{tot,z}=0\rangle$

Quindi $L_x|\psi_f\rangle = 0$ e $O^2|\psi_f\rangle = 4S_x^{e2}|\psi_f\rangle$

Per stati di spin 1/2 $S_x^{e2} = \frac{\hbar^2}{4}$ quindi $O^2|\psi_f\rangle = \hbar^2|\psi_f\rangle$

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Se $\psi_f(r) = \left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-r/a_0} \,|S_{tot}=0, S_{tot,z}=0\rangle$

$\psi_f'(r) = \chi^{3/2}\left(\frac{1}{\pi a_0^3}\right)^{1/2} e^{-\chi r/a_0} \,|S_{tot}=0, S_{tot,z}=0\rangle$

$\langle\psi_f'|\psi_f\rangle = \chi^{3/2}\frac{1}{\pi a_0^3}\int d^3r\, e^{-(1+\chi)r/a_0}$ Cambio variabile : $\frac{(1+\chi)r}{2} \to r$

$= \chi^{3/2}\left(\frac{2}{1+\chi}\right)^3 \frac{1}{\pi a_0^3}\int d^3r\, e^{-2r/a_0}$

$= \chi^{3/2}\left(\frac{2}{1+\chi}\right)^3 \langle\psi_f|\psi_f\rangle = \frac{8\chi^{3/2}}{(1+\chi)^3}$

$P = 1 - |\langle\psi_f'|\psi_f\rangle|^2 = 1 - \frac{64\chi^3}{(1+\chi)^6}$

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$E_{fond} = -E_I - \frac{3\alpha}{4} + \beta\,\langle\psi_f|S_z^e S_z^p|\psi_f\rangle$

$= -E_I - \frac{3\alpha}{4} + \beta\,\langle S_{tot}=0, S_{tot,z}=0|\,S_z^e S_{z}^p\,|S_{tot}=0, S_{tot,z}=0\rangle$

$= -E_I - \frac{3\alpha}{4} - \beta\frac{\hbar^2}{4}$