Domanda 1

Se $H_0 = \frac{p^2}{2\mu} + \frac{\mu\omega^2}{2}(x^2+y^2) + \frac{\mu(2\omega)^2}{2} z^2$

$$H_0 |n_x n_y n_z\rangle = \left[\hbar\omega(n_x+n_y+1) + 2\hbar\omega\left(n_z+\frac{1}{2}\right)\right] |n_x n_y n_z\rangle$$

Dato che $\psi_n(x) \xrightarrow{P} \psi_n(x)(-1)^n = \psi(-x)$ sotto parità

[OSCILLATORE UNIDIMENSIONALE]

$$P|n_x n_y n_z\rangle = (-1)^{n_x+n_y+n_z} |n_x n_y n_z\rangle \quad \text{con} \quad P\vec{r}P^\dagger = -\vec{r}$$

Spettro di H_0 fino a $E = 4\hbar\omega$

$E = 4\hbar\omega$ ——— $\underbrace{|200\rangle, |110\rangle, |020\rangle}_{\text{pari sotto } P}$, $|001\rangle$ ← dispari

$E = 3\hbar\omega$ ——— $|100\rangle, |010\rangle$ ← dispari sotto P

$E = 2\hbar\omega$ ——— $|000\rangle$ ← pari sotto P

N.B.: H_0 non è centrale: NON si possono classificare gli stati usando il momento angolare

Spettro della teoria completa usando il princ. Pauli

$E = 4\hbar\omega + \hbar\alpha$ $\quad |001\rangle \chi_1^{\pm 1}$ deg. 2

$E = 4\hbar\omega$ $\quad \begin{cases} |001\rangle \chi_1^0 \\ |200\rangle \chi_0^0, |110\rangle \chi_0^0, |020\rangle \chi_0^0 \end{cases}$ deg. 4

$E = 3\hbar\omega + \hbar\alpha$ $\quad |100\rangle \chi_1^{\pm 1}, |010\rangle \chi_1^{\pm 1}$ deg. 4

$E = 3\hbar\omega$ $\quad |100\rangle \chi_1^0, |010\rangle \chi_1^0$ deg. 2

$E = 2\hbar\omega$ $\quad |000\rangle \chi_0^0$

Domanda (2)

$$\phi(t) = \frac{1}{\sqrt{2}}\left(e^{-3i\omega t}|100\rangle + e^{-5i\omega t}|210\rangle\right)\chi_0^1$$
$$= \phi(r,t)\,\chi_0^1$$

(a) <u>NO</u>

(b) $\langle \phi t | (S_{Ax} + S_{Bx})^2 | \phi t \rangle =$ integrando la parte spaziale

$= \langle 1 0 | S_x^2 | 1 0 \rangle \quad =$ [QUI $|1 m\rangle = \chi_1^m$, $\bar{S} = \bar{S}_A + \bar{S}_B$]

$= | S_x |1, 0\rangle |^2$ [S_x è hermitiano]

$= \frac{1}{4} | S_+ |1,0\rangle + S_- |1,0\rangle |^2$

Ora $S_\pm |1 0\rangle = \sqrt{2}\hbar |1 \pm 1\rangle$. Quindi

$\langle \quad \rangle = \frac{1}{4} \left| \sqrt{2}\hbar |1 + 1\rangle + \sqrt{2}\hbar |1 - 1\rangle \right|^2 = \hbar^2$

Calcolo equivalente

$\langle 1 0 | S_x^2 | 1 0 \rangle = \frac{1}{4} \langle 1 0 | (S_+ + S_-)^2 | 1 0 \rangle$

$= \frac{1}{4} \left[\langle 1 0 | S_+ S_- + S_- S_+ | 1 0 \rangle \right] =$ $\left[\begin{array}{l} S_+^2 |1 0\rangle = 0 \\ S_-^2 |1 0\rangle \; 0 \end{array} \right]$

$= \frac{1}{4} \left[2\hbar^2 + 2\hbar^2 \right] = \hbar^2$

(c) Tenendo conto che $\langle E|x|E\rangle = 0$ per gli autostati dell'oscillatore UNIDIMENSIONALE

$$\langle\phi(t)|xy|\phi(t)\rangle =$$

$$= \frac{1}{2}\Big(\langle 100|xy|210\rangle e^{-2i\omega t} + \langle 210|xy|100\rangle e^{2i\omega t}\Big)$$

$$= \frac{1}{2}\Big(\langle 1|x|2\rangle\langle 0|y|1\rangle e^{-2i\omega t} + \text{complesso coniugato}\Big)$$

Se $|n\rangle = \frac{1}{\sqrt{n!}}(a^+)^n|0\rangle \qquad x = \sqrt{\frac{\hbar}{2\mu\omega}}(a+a^+)$

$$\langle 1|x|2\rangle = \sqrt{\frac{\hbar}{\mu\omega}} \qquad \langle 1|y|0\rangle = \sqrt{\frac{\hbar}{2\mu\omega}}$$

$$\langle\phi(t)|xy|\phi(t)\rangle = \frac{\hbar}{\mu\omega\sqrt{2}}\cos 2\omega t$$

Se $|n\rangle = \frac{1}{\sqrt{n!}}(\eta^+)^n|0\rangle \qquad x = -i\sqrt{\frac{\hbar}{2\mu\omega}}(\eta^+ - \eta)$

$$\langle 1|x|2\rangle = i\sqrt{\frac{\hbar}{\mu\omega}} \qquad \langle 1|y|0\rangle = -i\sqrt{\frac{\hbar}{2\mu\omega}}$$

$$\langle\phi(t)|xy|\phi(t)\rangle = -\frac{\hbar}{\mu\omega\sqrt{2}}\cos\omega t$$

DOMANDA 3: (a) $\beta = \left(\frac{m\omega}{\pi\hbar}\right)^{3/4}$ (b) NO

(c) $$\psi_{fond} = \underbrace{2^{1/4}\left(\frac{\mu\omega}{\pi\hbar}\right)^{3/4}}_{A}\exp\left[-\frac{\mu\omega}{2\hbar}(x^2+y^2+2z^2)\right]$$

$$\langle\psi,\psi_f\rangle = \beta A\int dx\,dy\,dz\,\exp\left[-\frac{\mu\omega}{2\hbar}(x^2+y^2+2z^2)\right]\exp\left(-\frac{m\omega}{2\hbar}r^2\right)$$

con $r^2 = x^2+y^2+z^2$

$$\langle\psi|\psi_f\rangle = \frac{2\sqrt{2}}{3}$$